Amplitude Variations in Pulsating Yellow Supergiants

John R. Percy
Rufina Y. H. Kim
Department of Astronomy and Astrophysics, University of Toronto, Toronto, ON M5S 3H4, Canada

Received June 12, 2014; revised July 24, 2014; accepted July 24, 2014

Abstract It was recently discovered that the amplitudes of pulsating red giants and supergiants vary significantly on time scales of 20–30 pulsation periods. Here, we analyze the amplitude variability in 29 pulsating *yellow* supergiants (5 RVa, 4 RVb, 9 SRd, 7 long-period Cepheid, and 4 yellow hypergiant stars), using visual observations from the AAVSO International Database, and Fourier and wavelet analysis using the AAVSO's VSTAR package. We find that these stars vary in amplitude by factors of up to 10 or more (but more typically 3–5), on a mean time scale (L) of 33 ± 4 pulsation periods (P). Each of the five sub-types shows this same behavior, which is very similar to that of the pulsating red giants, for which the median L/P was 31. For the RVb stars, the lengths of the cycles of amplitude variability are the same as the long secondary periods, to within the uncertainty of each.

1. Introduction

The amplitudes of pulsating stars are generally assumed to be constant. Those of multi-periodic pulsators may appear to vary because of interference between two or more modes, though the amplitudes of the individual modes are generally assumed to stay constant. Polaris (Arellano Ferro 1983) and RU Cam (Demers and Fernie 1966) are examples of "unusual" Cepheids which have varied in amplitude. The long-term, cyclic changes in the amplitudes of RR Lyrae stars—the Blazhko effect—are an ongoing mystery (Kolenberg 2012), period-doubling being a viable explanation. There are many reports in the literature of Mira stars which have varied systematically in amplitude.

Percy and Abachi (2013) recently reported on a study of the amplitudes of almost a hundred pulsating red giants. They found that, in 59 single-mode and double-mode SR variables, the amplitudes of the modes varied by factors of 2–10 on time scales of 30–45 pulsation periods, on average. Percy and Khatu (2014) reported on a study of 44 pulsating red *supergiants*, and found similar behavior: amplitude variations of a factor of up to 8 on time scales of 18 pulsation periods, on average.

In the present paper, we study the amplitudes of 29 pulsating *yellow* supergiants, including 9 RV Tauri (RV) stars, 9 SRd stars, 7 long-period

Cepheids, and 4 yellow hypergiants. RV Tau stars show alternating deep and shallow minima (to a greater or lesser extent). RVa stars have constant mean magnitude. RVb stars vary slowly in mean magnitude; they have a "long secondary period." SRd stars are semiregular yellow supergiants. Actually, there seems to be a smooth spectrum of behavior from RV to SRd and possibly to long-period Population II Cepheids (Percy *et al.* 2003).

Population I (Classical) Cepheids and yellow hypergiants differ from RV and SRd stars in that they are massive, young stars, whereas the latter two classes are old, lower-mass stars. Classical Cepheids tend to have shorter periods in part because the period is inversely proportional to the square root of the mass. We did not analyze short-period Cepheids because, with visual observations, it is necessary to have much denser coverage (a large number of observations per period) in order to beat down the observational error, which is typically 0.2–0.3 magnitude per observation. Bright short-period Cepheids such as δ Cep should have enough photoelectric photometry, over time, to detect amplitude variations if they exist. We recommend that such a study be carried out. Indeed, Derekas *et al.* (2012) analyzed 600 days of ultra-precise Kepler photometry of the 4.9-day Cepheid V1154 Cyg, and found small, cyclic variations in amplitude on a time scale of tens of pulsation periods. In the present project, we analyzed the prototype Population II Cepheid, W Vir, but the period is short (17.27 days), and the data are sparse, so the results are not very meaningful.

The periods of the yellow hypergiants are poorly defined, partly because the pulsation is semiregular at best, and partly because the light curves are affected by the heavy mass loss and occasional "eruptions" in these stars (e.g. Lobel *et al.* 2004). Furthermore, the periods are so long that the number of cycles of amplitude variation is very poorly determined.

2. Data and analysis

We used visual observations from the AAVSO International Database of the yellow supergiant variables listed in Table 1. See "Notes on Individual Stars," and the last two columns in Table 1 for remarks on some of these. Our data extend for typically 10,000–30,000 days; not all the stars have the same length of dataset. Percy and Abachi (2013) discussed some of the limitations of visual data which must be kept in mind when analyzing the observations and interpreting the results. In particular: some of the stars have pronounced seasonal gaps in the data, which can produce "alias" periods and cause some difficulty in the wavelet analysis.

The data, extending over the range of Julian Date given in Table 1 were analyzed with the AAVSO's VSTAR time-series analysis package (Benn 2013), especially the Fourier (DCDFT) analysis and wavelet (WWZ) analysis routines. The JD range began where the data were sufficiently dense for analysis. The DCDFT routine was used to determine the best period for the JD range used.

It was invariably in good agreement with the literature period; in any case, the results are not sensitive to the exact value of period used. For the RV stars, we used the dominant period: either the "half" period—the interval between adjacent minima—or the "full" period, the interval between deep minima. We found that, whichever of these two periods we used, the value of L/P was the same to within the uncertainty.

Note that, in this paper, we use the term "amplitude" in a general way, to denote the strength of the variation. We actually measure the *semi-amplitude*—the coefficient of a sine curve which would fit the data in the DCDFT analysis; this is what is given in the figures and tables. The full amplitude or range would be the difference between maximum and minimum magnitude.

For the wavelet analysis, the default values were used for the decay constant c (0.001) and time division Δt (50 days). The results are sensitive to the former, but not to the latter. For the WWZ analysis: around each of the adopted periods, we generated the amplitude versus JD graph, and determined the range in amplitude, and the number (N) of cycles of amplitude increase and decrease, as shown in Figures 1 through 10. N can be small and ambiguous (see below), so it is not a precise number.

For a few stars with slow amplitude variations, we checked and confirmed the amplitude variability by using the DCDFT routine to determine the amplitude over sub-intervals of the range of JD chosen. For a few stars, we also repeated the analysis using c = 0.003 and 0.005 as well as 0.001. The results did not change though, for one or two of the stars, the amplitude-JD curves were slightly more scattered when c = 0.005.

3. Results

Table 1 lists the results. It gives the name of the star, the type of variability, the adopted period P in days, the range of JD of the observations, the maximum and minimum amplitude, the number N of cycles of amplitude increase and decrease, the average length L in days of the cycles as determined from the JD range and N, the ratio L/P, a rough measure D of the average density of the light curve relative to the period (1 = densest, 3 = least dense), and a rough measure R of the robustness or reliability of the amplitude versus JD curve (1 = most reliable, 3 = least reliable). The least reliable curves have gaps, much scatter, and are generally the ones that are least dense. The Cepheids tended to be less reliable because of their shorter periods, lower density, and smaller amplitudes. They also tend to be less well-observed visually because observers assume that they are best observed photoelectrically. The yellow hypergiants are moderately reliable in the sense that the data are dense and the amplitude-time graphs are well-defined, but the number of cycles is small, the amplitude is small, and the variability is inherently semi-regular at best. Note that the stars in Table 1 have a wide range of amplitudes and amplitude ranges.

Table 1. Amplitude variability of pulsating yellow supergiants.

Star	*Type*	*P(d)*	*JD Range*	*A Range*	*N*	*L/P*	*D*	*R*
V1302 Aql	YHG	359	2442542–2455368	0.02–0.09	1.5	23.8	2	2
Z Aur	SR/SRd	110.40	2423000–2456651	0.20–0.76	15	20.3	1	1
AG Aur	SRd	96.13	2441000–2456600	0.15–0.78	11	14.8	2	2
U Car	DCEP	38.83	2443617–2456627	0.33–0.63	9	37.2	1	2
IW Car	RVb	71.96	2446037–2456646	0.05–0.24	8	18.4	1	2
l Car	DCEP	35.54	2426096–2456609	0.22–0.42	12	71.5	1	3
V509 Cas	YHG	878	2446923–2456719	0.03–0.06	0.25	44.6	2	2
ρ Cas	YHG	659	2433000–2456723	0.02–0.07	1.35	26.7	1	2
V766 Cen	YHG	871	2446933–2456709	0.09–0.15	0.40	28.1	1	2
TZ Cep	SRd	82.44	2451426–2456635	0.33–0.76	2.2	28.7	2	1
AV Cyg	SRd	87.97	2429012–2456630	0.11–0.57	9	34.9	1	1
DF Cyg	RVb	24.91	2441000–2456600	0.20–0.86	20	31.3	2	1
SU Gem	RVb	24.98	2446000–2456250	0.00–1.35	15	27.4	3	3
UU Her	SRd	44.93	2432000–2456622	0.02–0.18	15	36.5	2	2
AC Her	RVa	37.69	2435500–2456600	0.28–0.49	12.5	44.8	1	2
DE Her	SRd	173.10	2442000–2456622	0.13–0.64	1.5	56.3	1	1
SX Her	SRd	103.50	2425000–2456631	0.08–0.44	6.5	47.0	1	1
RS Lac	SRd	237.57	2427592–2456635	0.35–1.02	3.5	34.9	1	1
SX Lac	SRd	195.48	2446000–2456282	0.10–0.17	3.75	14.0	2	1
T Mon	DCEP	27.03	2441500–2456400	0.28–0.53	15	36.7	1	2
TT Oph	RVa	30.51	2427946–2456615	0.25–0.76	39	24.1	1	2
UZ Oph	RVa	43.71	2445500–2456626	0.20–0.70	9.5	26.8	1	1

Table continued on next page

Table 1. Amplitude variability of pulsating yellow supergiants, cont.

Star	*Type*	*P(d)*	*JD Range*	*A Range*	*N*	*L/P*	*D*	*R*
TX Per	RVa	76.38	2427964–2456654	0.12–0.75	7.5	50.1	2	1
X Pup	DCEP	25.96	2434287–2456416	0.15–1.04	15	56.8	3	3
AI Sco	RVb	35.76	2445000–2455750	0.15–1.10	11	27.3	2	2
W Vir	CWA	17.27	2425000–2456458	0.16–1.29	41	16.3	2	3
S Vul	DCEP	68.30	2439626–2456558	0.10–1.20	11	22.5	3	2
V Vul	RVa	76.31	2446000–2456649	0.20–0.35	6.5	21.5	1	1
SV Vul	DCEP	44.98	2439622–2456637	0.33–0.54	13.5	28.0	1	1

The maximum and minimum amplitudes were determined with due regard to the scatter in the amplitude versus JD curves and the number of points defining each maximum or minimum. The process of counting the number of cycles was somewhat subjective, but was consistent, having been done by co-author Kim using Percy and Abachi (2013) and Percy and Khatu (2014) as a model. Figures 1 and 2 show examples of the process for the RVa star AC Her and the SRd star SX Her and its uncertainty. They also show the difference between the amplitude versus JD curves for a shorter-period star and a longer-period one. One could argue for adding or removing one or two cycles in each case, but the number should be reliable to ±10–20 percent. Figures 3–10 show examples of the amplitude versus JD curves for other representative stars.

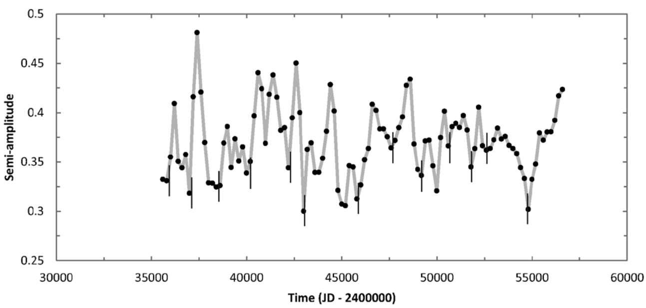

Figure 1. Amplitude versus Julian Date for the RVa star AC Her, showing where we assume the minima to be. We count 12.5 cycles in a JD range of 21100 days, giving a cycle length L of 1688 days, but the uncertainty in doing this is apparent from the graph; one could argue for the addition or removal of one or two maxima or minima. The dominant pulsation period is the "half" period, 37.69 days. Compare this diagram with that for SX Her, which has a longer period.

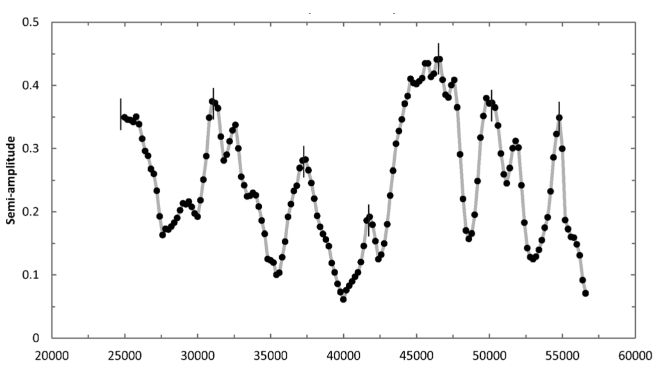

Figure 2. Amplitude versus Julian Date for the SRd star SX Her, showing where we assume the maxima to be. We count 6.5 cycles in a JD range of 31631 days, giving a cycle length L of 4866 days. The uncertainty in doing this is apparent from the graph, but it is less than for AC Her (Figure 1). The pulsation period is 103.50 days.

Table 2. Summary statistics: amplitude variability of pulsating yellow supergiants.

Type	*Mean P (SD)*	*Mean L/P (SEM)*	*Mean D*	*Mean R*
Cepheids	41.09 (14.45)	38.46 (7.38)	1.71	2.29
RVa	52.92 (21.89)	33.45 (5.83)	1.20	1.40
RVb	39.40 (22.30)	26.11 (2.72)	2.00	2.00
SRd	125.72 (2.30)	31.94 (4.75)	1.44	1.22
Hypergiants	700:	31:	1.5	2:
Robust	104.73 (65.12)	32.82 (3.67)	1.33	1.00

Table 2 presents summary statistics for the sub-groups of stars: Cepheids, RVa, RVb, SRd, and hypergiant. Note that the mean L/P is the same, within the standard error of the mean (SEM), for all groups, and is the same as the median L/P (31) for pulsating red giants (Percy and Abachi 2013). The values for the hypergiants are very uncertain, so we have listed only approximate numbers. The last line ("Robust") refers to the stars whose amplitude versus JD curves appear to be the most dependable. Given the uncertainty in the value of L for each star, the uniformity of the *average* values of L/P is most significant.

3.1. Notes on individual stars

These notes are given in the same order as the stars are listed in Table 1. See also the last two columns in Table 1 for information about the denseness of the light curves, and the robustness of the amplitude versus JD curves.

SU Gem: The seasonal gaps are very conspicuous.

AC Her: Figure 1. The increase in amplitude since JD 2455000 is confirmed by the AAVSO photoelectric photometry.

Z Aur: Figure 5. This star shows periods of 112 and 135 days, and switches between them (Lacy 1973). There is some evidence that the amplitude of pulsation decreases before a switch takes place. Period switches occur at the following Julian Dates: 2428000 (112 to 135 days), 2439000 (135 to 112 days), 2443000 (112 to 135 days), and 2448500 (135 to 112 days), with the middle two switches being less distinct.

TZ Cep: Figure 6. The data are sparse since JD 2454200.

DE Her: This star shows 1.5 cycles of a long secondary period, but is classified as SRd rather than RVb.

UU Her: This star switches between periods of 45–46 and 72 days (Zsoldos and Sasselov 1992).

RS Lac: Figure 7. The amplitude variation is apparent from the light curve.

SX Lac: Figure 4. The data are initially sparse.

S Vul: The middle of the dataset is sparse.

V509 Cas: Later in the visual dataset, the dominant period is 259 days. The photoelectric V data, however, show periods between 350 and 500 days.

V1302 Aql: The period is suspiciously close to one year. Furthermore: the data are sparse and the amplitude is small.

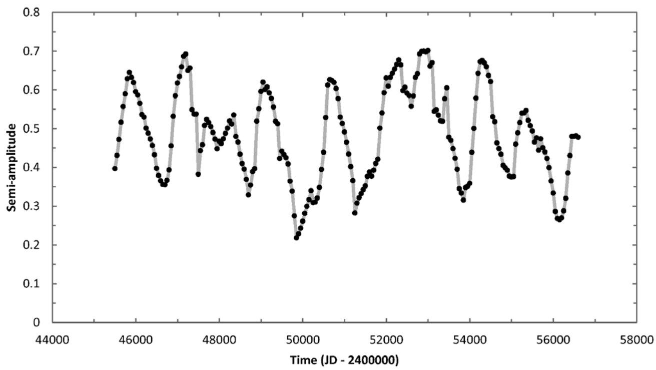

Figure 3. Amplitude versus Julian Date for the RVa star UZ Oph. We count 9.5 cycles. The curve is well-defined. The dominant pulsation period is the "half" period, 43.71 days.

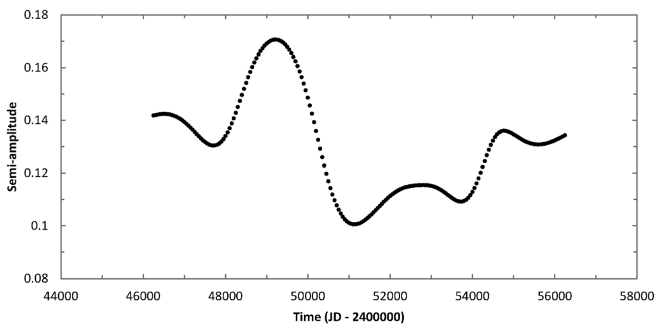

Figure 4. Amplitude versus Julian Date for the SRd star SX Lac. We count 3.75 cycles. The pulsation period is 195.48 days. Compare this graph with e.g. Figure 3 for UZ Oph, a shorter-period star, but note that the amplitude of SX Lac is small, and that presumably adds to the uncertainty in determining N.

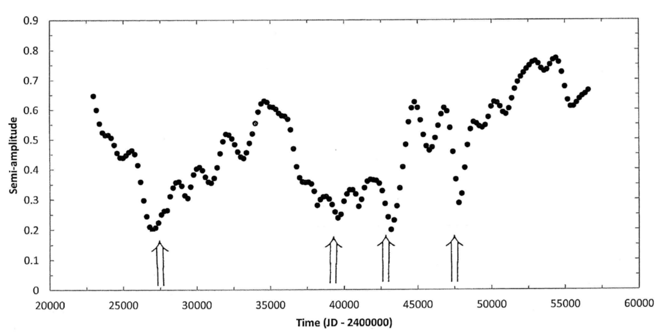

Figure 5. Amplitude versus Julian Date for the SRd star Z Aur. We count 15 cycles. This star is unusual in that its dominant period switches between about 112 and 135 days. The amplitude tends to decrease before period switches, which occured around JD 2428000 (112 to 135 days), 2439000 (135 to 112 days), 2443000 (112 to 135 days), and 2448500 (135 to 112 days), with the middle two switches being less distinct.

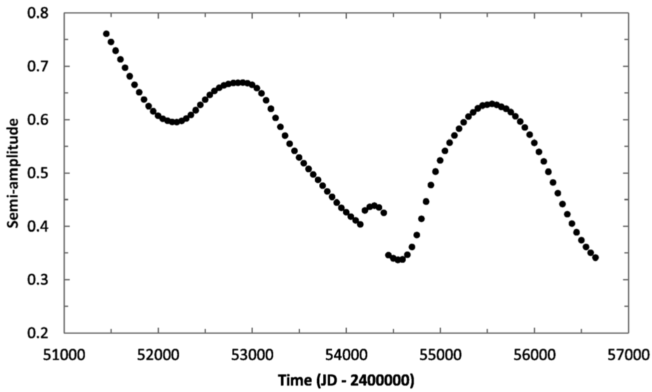

Figure 6. Amplitude versus Julian Date for the SRd star TZ Cep. We count 2.2 cycles. The pulsation period is 82.44 days.

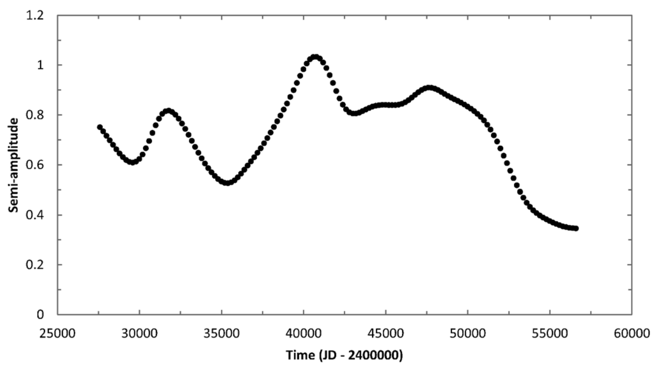

Figure 7. Amplitude versus Julian Date for the SRd star RS Lac. We count 3.5 cycles. The pulsation period is 237.57 days.

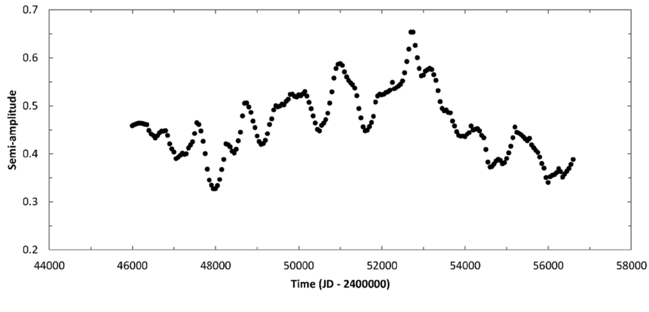

Figure 8. Amplitude versus Julian Date for the Cepheid U Car. We count 9 cycles. There is a slow change in amplitude, as well as the rapid ones. The pulsation period is 38.83 days. The rapid changes in amplitude are relatively small, suggesting that the mechanism which causes them is not dominant in this star.

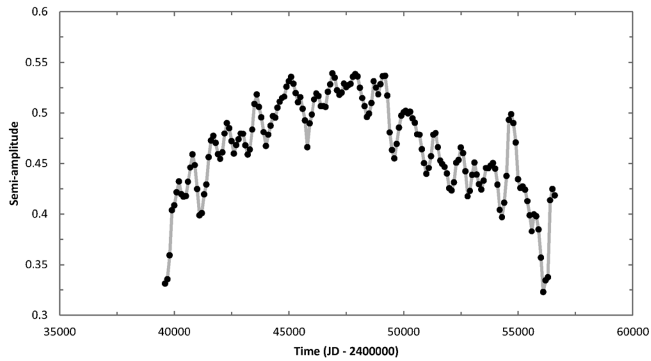

Figure 9. Amplitude versus Julian Date for the Cepheid SV Vul. We count 13.5 cycles. These are small and rapid, and therefore not well defined by our limited visual observations. There is also a slow change in amplitude, with the amplitude being smaller at the beginning of the observing period, larger in the middle, and smaller again at the end. The pulsation period is 44.98 days. The rapid changes in amplitude are relatively small, suggesting that the mechanism which causes them is not dominant in this star.

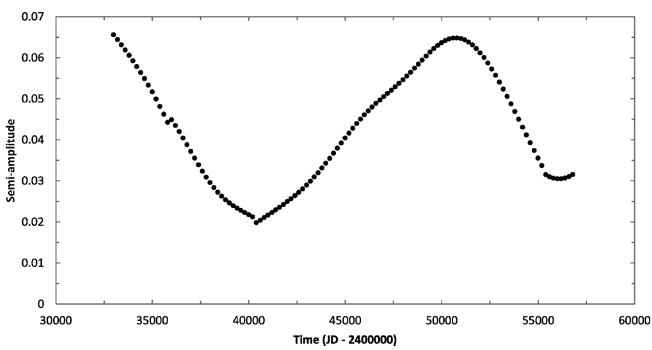

Figure 10. Amplitude versus Julian Date for the yellow hypergiant ρ Cas. We count 1.35 cycles, though this is obviously very uncertain—even more so for the other yellow hypergiants. The adopted pulsation period is 659 days.

4. Discussion

We have found that almost all of the pulsating yellow supergiants that we have studied vary in pulsation amplitude by a factor of up to 10 on a time scale of about 33 ± 4 pulsation periods. The behavior is similar in each of the subtypes of variables, and that behavior is similar to that of pulsating red giants (Percy and Abachi 2013). In particular: the RV Tauri variables showed similar L/P to the other types, whether the half-period or the full period was dominant. These results were pleasantly surprising to us, as we had no *a priori* reason to think that these stars would show amplitude variations or, if so, that these would be similar to those in red giants and supergiants. In some cases, however, the amplitude variation in these stars is visible in the light curve.

We note that, for the RVb stars, the lengths of the cycles of amplitude variability are the same as the lengths of the long secondary periods, within the uncertainties of each. For SU Gem, L = 683, LSP = 682; for IW Car: L = 1326, LSP = 1430; for DF Cyg: L = 780, LSP = 784; for AI Sco: L = 977, LSP = 975, the units being days in each case. Percy (1993) noted that, during the long secondary minima in the RVb star U Mon, the pulsation amplitude was low. This coincidence between L and LSP may help to elucidate the cause of both the RVb phenomenon and the amplitude variation.

Percy and Abachi (2013) proposed two possible explanations for the amplitude variation in pulsating red giants: (i) the rotation of a star with large inhomogeneities in its photosphere; and (ii) stochastic excitation and decay of pulsations, driven by convection (this possibility was suggested to us by Professor Tim Bedding). Red giants and supergiants are highly convective, and there is evidence (e.g. Kiss *et al.* 2006, Xiong and Deng 2007) that the convection interacts with the pulsation. Cepheid pulsations are excited by the kappa (opacity) mechanism; hydrodynamic models (e.g. Stobie 1969) show that the pulsation amplitude grows until the pulsational energy generation is balanced by dissipation. As for the amplitude *variations*, since yellow supergiants are not expected to show large inhomogeneities in their photospheres, stochastic excitation and decay is the more likely explanation. All of the stars in our sample are cooler than the sun (their (B–V) colors range from +0.9 to +1.8) so they all have significant external convection zones (though some of the stars, especially the yellow hypergiants, may be reddened by dust). We note that, in the long-period Cepheids U Car and SV Vul (Figures 8 and 9), the amplitude fluctuations are relatively small, but there are also slow changes in amplitude as well as the small, rapid ones. Derekas *et al.* (2012) also raised the possibility that convective processes might cause the period and light-curve fluctuations which they observed in V1154 Cyg. In addition to the possibility of stochastic excitation, these stars are subject to possible non-linear effects such as period doubling and chaos (Buchler and Kovacs 1987; Fokin 1994; Buchler *et al.* 1996; Buchler *et al.* 2004).

Amplitude variations complicate the study of these stars in the sense that, to compare photometric behavior with other types of behavior—spectroscopic, for instance—the observations must be made within a few pulsation periods of each other. The AAVSO provides an important service by monitoring many of these stars.

5. Conclusions

We have studied the amplitude variation in 29 pulsating yellow supergiants of several types: RV Tauri stars (RVa and RVb), SRd stars, long-period Cepheids, and hypergiants. In each case, we find amplitude variations of a factor of up to 10 (but more typically 3–5) on a time scale of 33 pulsation periods. The behavior is similar for each type of star, and is similar to that found by Percy and Abachi (2013) in pulsating red giants.

6. Acknowledgements

This project would not have been possible without the efforts of the hundreds of AAVSO observers who made the observations which were used in this project, and the AAVSO staff who processed and archived the measurements and made them publicly available, and the team which developed the VSTAR package and made it user-friendly and publicly available. We thank the University of Toronto Work-Study Program for financial support, and Professors Tim Bedding and Laszlo Kiss for reading and commenting on a draft of this paper. Author JRP thanks author RYHK, an undergraduate major in Ecology and Evolutionary Biology who, with no previous background in Astronomy, organized and carried out this project so professionally. This project made use of the SIMBAD database, which is operated by CDS, Strasbourg, France.

References

Arellano Ferro, A. 1983, *Astrophys. J.*, **274**, 755.

Benn, D. 2013, VSTAR data analysis software (http://www.aavso.org/node/803 and www.aavso.org/vstar-overview).

Buchler, J. R., Kolláth, Z., and Cadmus, R. R., Jr. 2004, *Astrophys. J.*, **613**, 532.

Buchler, J. R., Kolláth, Z., Serre, T., and Mattei, J. A. 1996, *Astrophys. J.*, **462**, 489.

Buchler, J. R., and Kovács, G. 1987, *Astrophys. J., Lett. Ed.*, **320**, 57.

Demers, S., and Fernie, J. D. 1966, *Astrophys. J.*, **144**, 440.

Derekas, A., *et al.* 2012, *Mon. Not. Roy. Astron. Soc.*, **425**, 1312.

Fokin, A. B. 1994, *Astron. Astrophys.*, **292**, 133.

Kiss, L. L., Szabó, G. M., and Bedding, T. R. 2006, *Mon. Not. Roy. Astron. Soc.*, **372**, 1721.

Kolenberg, K. 2012, *J. Amer. Assoc. Var. Star Obs.*, **40**, 481.

Lacy, C. H. 1973, *Astron. J.*, **78**, 90.

Lobel, A., Stefanik, R. P., Torres, G., Davis, R. J., Ilyin, I., and Rosenbush, A. E. 2004, in *Stars as Suns: Activity, Evolution, and Planets*, eds. A. K. Dupree and A. O. Benz, IAU Symp. 219, Astronomical Society of the Pacific, San Francisco, 903.

Percy, J. R. 1993, in *Non-Linear Phenomena in Stellar Variability* (*Astrophys. Space Sci.*, **210**), eds. M. Takeuti and J.-R. Buchler, IAU Collq. 134, Kluwer Academic Publishers, Dordrecht, The Netherlands, 123.

Percy, J. R., and Abachi, R. 2013, *J. Amer. Assoc. Var. Star Obs.*, **41**, 193.

Percy, J. R., Hosick, J., and Leigh, N. W. C. 2003, *Publ. Astron. Soc. Pacific*, **115**, 59.

Percy, J. R., and Khatu, V. 2014, *J. Amer. Assoc. Var. Star Obs.*, **42**, 1.
Stobie, R. S. 1969, *Mon. Not. Roy. Astron. Soc.*, **144**, 461.
Xiong, D. R., and Deng, L. 2007, *Mon. Not. Roy. Astron. Soc.*, **378**, 1270.
Zsoldos, E., and Sasselov, D. D. 1992, *Astron. Astrophys.*, **256**, 107.