PERIODS OF EO AND FV COMAE BERENICES AND THE FUNCTIONAL FORMS OF THEIR O-C DIAGRAMS

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Abstract

New elements are given for the RRab Lyrae variables EO Comae Berenices and FV Comae Berenices. The statistical significance of higher order terms in a polynomial least squares fit is discussed.

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1. Introduction

Magnitude estimates of EO Comae Berenices and FV Comae Berenices were made from over 450 plates taken at the Maria Mitchell Observatory (MMO), interpolating between nearby comparison stars. These data were grouped according to calendar year, and each year's light curve was compared to a mean light curve to determine the phase shift, or O-C value. The error bars given for each O-C value are the maximum ranges in phase for which the years' O-C light curves still fit the mean light curve. They correspond to an approximately 95% confidence level. These O-C values were then plotted versus date. Least squares methods were used to determine the best polynomial to fit the data, with the weight of each point proportional to the inverse square of the error. The elements of the variable were then determined from the equation of the O-C versus date curve. All dates in this paper are heliocentric.

2. FV Comae Berenices

FV Com is a 14th magnitude RR Lyr variable rediscovered by Keyes (1970). Its 1900 coordinates are R.A. = $12^h\ 33^m\ 02^s$; Dec = $+30^\circ\ 31'.2$. A finding chart is given in Figure 1. Using plates taken at MMO between 1964 and 1970, Keyes found the following elements:

$$JD_{(max)} = 2438887.686 + 0.472469\ E. \qquad (1)$$

I re-estimated the magnitudes on these plates in addition to more recent ones taken at MMO for a total of 381 plates taken between 1964 and 1985. The comparison magnitudes were derived from the Lick Observatory sequence RR3 (Kinman et al. 1966). The O-C diagram in Figure 2 was then constructed. The best line implied the elements:

$$\begin{aligned} JD(max) = &\ 2446173.642 + 0.4724752\ E + 4.0 \times 10^{-10}\ E^2. \qquad (2) \\ &\ \pm 0.008 \quad \pm 0.0000022 \quad \pm 1.4 \times 10^{-10} \end{aligned}$$

Is the squared term significant? Looking at Figure 2, it is hard to tell. However, realizing that points 1 and 15 have high error (and thus low weight) and that ignoring them will not change the computed curves much makes the points appear much more like a parabola. But there is a more statistically accurate method.

Pringle (1975) derived a form of the F-test specifically for testing the statistical significance of the squared term. His method calculates P, a number between 0 and 1, which gives the confidence level at which we can eliminate the assumption that the squared term is zero. A confidence level of at least 90% (P = 0.90) is usually required before the squared term is deemed significant. For the parabola in Figure 2, P = 0.99. That is, there is only a 1% probability that the deviations from the line are due to chance. Or, the mean error is significantly reduced using a parabola rather than a line, and there is a 99% probability that the squared term is not zero.

If a parabola works so well, would an extra, higher order term (a cubic) work better? How can the significance of the cubed term be tested? Pringle's statistic, λ, can be generalized to determine the significance of any higher order polynomial term. If S_2 is defined as the sum of squares of the residuals from the curve we are testing, in this case a cubic:

$$S_2 = \sum_1^n \left[(O-C) - (a_0 + a_1E + a_2E^2 + a_3E^3)\right]^2, \tag{5}$$

and S_1 is similarly defined for the next lower order curve (in this case a parabola), then λ is:

$$\lambda = \frac{(S_1 - S_2)/1}{S_2 / (n - m)} \tag{6}$$

where n is the number of points, m is the number of parameters involved in the higher order curve, and n-m is the number of degrees of freedom of the higher order curve. For a cubic, m = 4 (there are 4 coefficients). The difference between the degrees of freedom of the two curves is unity, so the confidence level P is then given by:

$$\lambda = F_p\ (1,\ n - m) \tag{7}$$

where the F-distribution is described by Brandt (1976), among others.

Using this method, a cubic and a measure of its significance (λ) were calculated for the points in Figure 2. The F-test revealed P = 0.38. That is, there is only a 38% probability that the cubed term is not zero. With this low probability, it must be concluded that the form of Figure 2 is fit best by a parabola, which implies the elements in equation (4) and a period increasing at the rate:

$$\beta = (+1.3 \pm 0.4) \times 10^{-6} \text{ cycles/year}. \tag{8}$$

2. EO Comae Berenices

EO Com is a 15th magnitude RR Lyr variable that was discovered in the Lick Observatory Survey of the North Galactic Pole by Kinman et al. (1966). Coordinates and a finding chart are given in Kinman et al., and their comparison magnitudes were used for the plates taken at MMO. EO Com is No. 36 in their list. They took 40 plates from 1961 to 1964 and derived the elements:

$$JD_{(max)} = 2437351.572 + 0.63211\ E. \tag{9}$$

Butler et al. (1979), also at Lick, took 16 more plates in 1977 and revised the period to:

$$P = (0.632090 \pm 0.000002) \text{ days}. \tag{10}$$

These data, along with 88 plates taken at MMO between 1980 and 1985, were used to construct the O-C diagram in Figure 3. The best line implies the following elements:

$$JD_{(max)} = \underset{\pm 0.008}{2446230.554} + \underset{\pm 0.0000010}{0.6320893}\ E. \tag{11}$$

The best parabola was calculated, and the significance of the squared term as described above was P = 0.70. This probability is too low to believe in the parabola. Hence, the line appears to be the best polynomial, implying a constant period. This can be verified by looking at Figure 3.

4. Conclusions

When searching for a functional form for an O-C diagram, it is good to keep in mind Occam's razor: the simplest one is probably the right one! The rate of evolution-caused period changes is very small (if even existent), and it is hard to detect changes in the rate of period change (i.e., a cubed term). Also, because of the slow rate of change, the best elements for future prediction tend to come from lines and parabolas. To believe in anything more complicated is to probably read more into the scattered data than is there.

5. Acknowledgements

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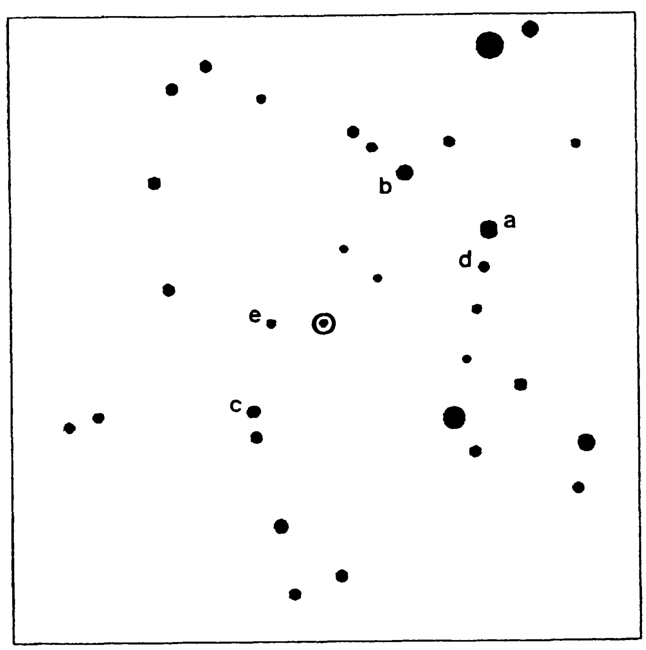

Figure 1. A finder chart for FV Com. North is up, and one side equals 30 arc seconds. The variable is indicated by a bullseye. The comparison magnitudes (photographic) are a = 13.3, b = 13.8, c = 14.3, d = 14.7, e = 15.1.

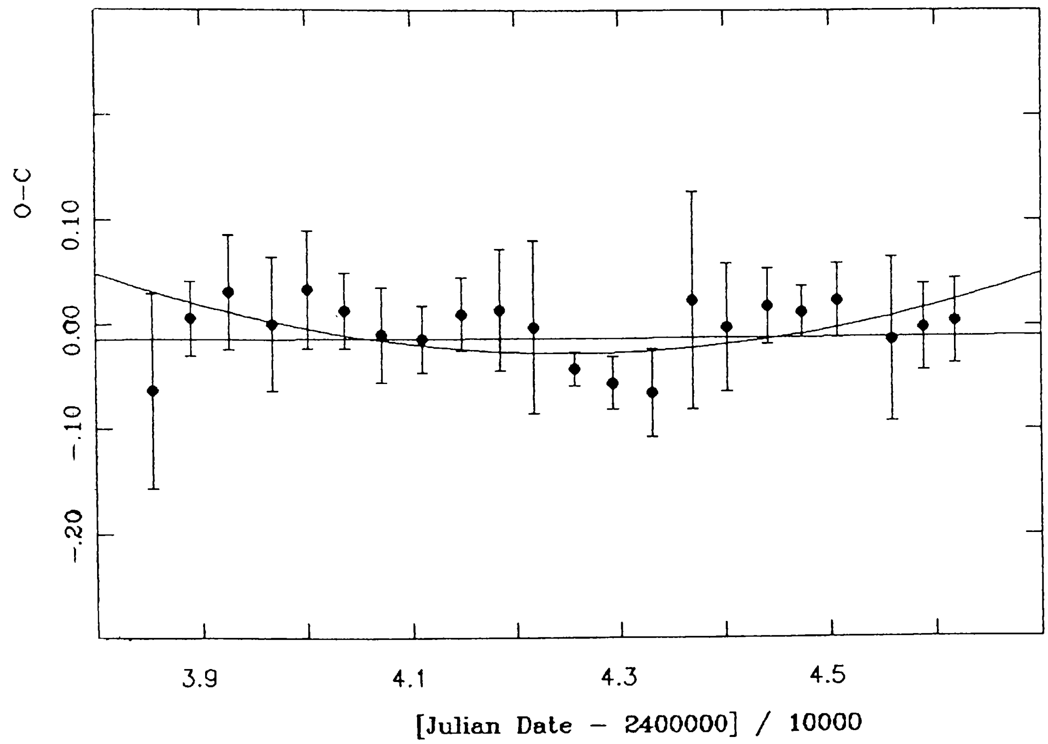

Figure 2. The O-C diagram for FV Com. The assumed elements are those in equation (2). All points are from MMO plates.

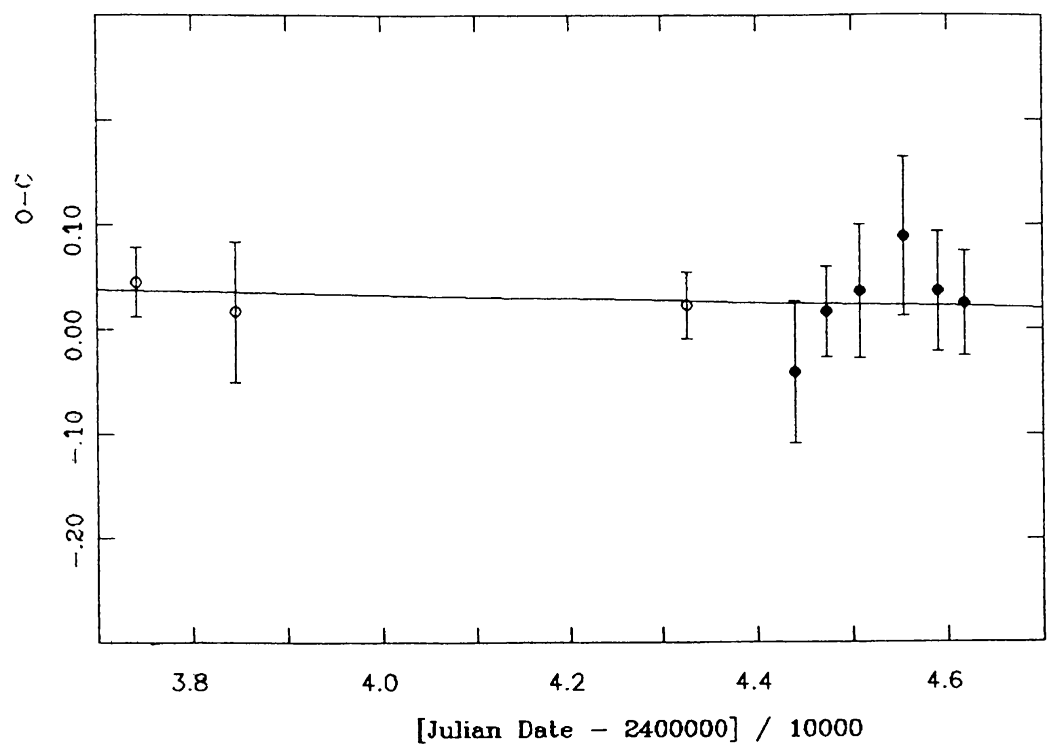

Figure 3. The O-C diagram for EO Com. The assumed elements are $JD_{(max)}$ = 2437351.572 + 0.632090 E. The filled circles are derived from MMO plates, and the unfilled circles are from Lick (Kinman et al.; Butler et al.).