V714 CYGNI: A STUDY IN THE EVOLUTION OF A BL HERCULIS STAR

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Abstract

An increasing period determined for V714 Cygni suggests evolutionary changes. A comparison with quantities derived from evolutionary models indicates a possible helium abundance and mass.

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Miller first discussed V714 Cygni as a Classical Cepheid variable (1953). After examining both Vatican and Harvard plates, he constructed a light curve of the variable star with data from the years 1926 to 1952. He then determined both the period and epoch of the star: namely, \( P = 1.8873679 \) days and \( JD_0 = 2432515.4872 \).

Harris (1985a) subsequently classified V714 Cyg as a Population II Cepheid because of this field star's large distance (700 pc) above the galactic plane. Specifically, Population II Cepheids like V714 Cyg with low luminosities and short periods (less than about three days) are referred to as BL Herculis stars, after the brightest field star of this type.

Theory about BL Her stars indicates that they should be evolving rapidly (Gingold 1976). Indeed, many evolve fast enough for observers to notice (e.g., Wehlau and Bohlender 1982; Christianson 1983; Provenca 1986). As they journey from the horizontal branch toward the asymptotic giant branch, they zigzag across the instability strip, at times expanding and cooling, and at other times shrinking and heating. Recall that the period of a variable is inversely proportional to the square-root of the mean density, and the density decreases as the star expands and cools. Thus, as the star cools and crosses to the red side of the instability strip, its density decreases and its period increases. Conversely, as the star heats up and turns blue, its density increases and its period decreases.

It was reasoned that if V714 Cyg is a BL Her star, then the star's present period should have changed from the period published by Miller. Peculiarly, Miller's period contains eight significant digits - surprising accuracy for a rapidly evolving star. The author therefore examined 986 Maria Mitchell Observatory (MMO) plates, spanning the years 1953 to 1988, to calculate new elements for the variable.

Stars 'b', 'c', 'e', and 'f' from Miller's paper (1953) were used for comparison. However, the author's magnitudes for the four stars differed from Miller's in three ways. First, Miller's magnitudes were given to two decimal places; for my purposes, such accuracy was unnecessary and the magnitudes were rounded so that, for example, 12.92 became 12.9 and 13.65 became 13.7. Secondly, Miller had accidentally interchanged the magnitudes of two sequence stars on his chart of the field: star 'b' was labeled 13.23 when it should have been 12.92, and star 'a' was labeled 12.92 when it should have been 13.23. Apparently, his published estimates of V714 Cyg were based upon the correct magnitudes and he confused the labeling of the two stars only in the published figure. The magnitudes were accordingly switched back for my own estimates. Third, iris photometry indicated that the magnitude for star 'f' was simply too faint; Miller's value, 15.08, was changed to 14.8.
For each of the 986 magnitude estimates, phases were calculated with Miller's elements. The data were then divided into three-year intervals and plotted as light curves. Thus twelve light curves, spanning the years 1953-1988, were plotted with data gathered from MMO plates. Similarly, the author constructed four light curves with Miller's magnitude estimates, each curve spanning a period of three years, together spanning the years 1941-1952. Although Miller included seven estimates dating before 1941, they were too few, too uncertain, and too infrequent to form a useful light curve.

Care was taken while incorporating Miller's data with my own. Because the magnitude of one of Miller's sequence stars had been corrected from 15.08 to 14.8, all of Miller's estimates were rescaled by linear interpolation. Nevertheless, there was a systematic difference between Miller's light curves and my own by about -0.3 magnitude.

\[(O-C)/P\] values for each of the sixteen light curves were determined. That is, the author found for each three-year light curve how far the maximum computed from trial elements deviated from the observed. A computer program fit each three-year light curve with an average light curve, minimizing the sum of the squares of the residuals (Belserene 1986). By adopting the brightest point on the average light curve as the maximum, the program found the phase of maximum of each three-year light curve, i.e., the value for \[(O-C)/P\].

With these sixteen values for \[(O-C)/P\], spanning the years 1941-1988, the author constructed an O-C graph (Figure 1). A parabola which was the best fit to the sixteen points, according to a least-squares solution, indicated the following new elements:

\[
\text{JD}_{\text{max}} = 2442092.235 + 1.8874572 \ E + 4.2 \times 10^{-9} \ E^2. \tag{1}
\]

\[
\pm 0.004 \pm 0.0000021 \pm 0.6 \times 10^{-9}
\]

These elements correspond to a period increasing by 1.6 ± 0.25 days per million years.

An effort was made to correlate the observations of V714 Cyg with theory about the evolution of BL Her stars. Gingold (1976) has constructed several grids of post-horizontal-branch evolutionary models assuming a range in either initial core mass or total mass on the horizontal branch. An average luminosity for V714 Cyg, determined with a period-luminosity relationship (Demers and Harris 1974) and a bolometric correction (Harris 1963), was found to be in general agreement with the luminosities of Gingold's models. Unfortunately, period-luminosity relationships suffer from an inherent uncertainty due to the width of the instability strip itself (Cox 1980); determining the luminosity in such a manner may wash out the resolution needed to distinguish between different models.

A more robust comparison between the observed properties of V714 Cygni with the evolutionary models was then undertaken. Following Gingold's example, the author reconstructed the fundamental blue edge of the instability strip from Tables 4, 5, 6, and 7 from the work of Tuggle and Iben (1972). For each mass and crossing of the instability strip given in Tables 1 and 2 of Gingold's paper, the periods of the blue and red edges of the strip and the rate of change of period in cycles per year were calculated.

The periods at the edges of the strip can be determined by first recalling that a star radiates like a blackbody; the luminosity is therefore dependent on the radius and the temperature by:

\[L = R^2 \times T^4. \tag{2}\]
where the variables are given in solar units. Recall also that the density is given by:

\[ d \sim M x R^{-3}, \]  

(3)

if the density, mass, and radius are also given in solar units. Lastly, the period-density relation is given by

\[ P = d^{-1/2} x Q, \]  

(4)

where Q is the pulsation constant.

Substituting equations (2) and (3) into equation (4) gives:

\[ P \sim (Q x L^{3/4})/(M^{1/2} x T^3). \]  

(5)

Gingold supplies the mass and luminosity for each of his models, and values for T and Q can be determined from the work of Tuggle and Iben. With equation (5), the period at both edges can therefore be determined for each crossing of the instability strip.

The rate of change of period in cycles per year is given by the equation:

\[ \text{rate of change} = (\Delta P)/(\Delta t x P), \]  

(6)

where \( \Delta P \) equals the period of the red edge minus the period of the blue edge (found from equation 5), \( \Delta t \) is Gingold's value for the time the star takes to cross the instability strip, and P is the average period across the strip.

Of Gingold's models for stars with an initial core mass of 0.4664 solar mass and a heavy element abundance of \( Z = 0.001 \), the best model found has a mass of 0.519 solar mass, a period between 1.3 and 2.0 days, a rate of change equal to \( 5.0 \times 10^{-9} \) cycle per year, and a helium envelope content of \( Y = 0.2 \). The star was crossing the instability strip for the third time.

Although some evidence suggests that \( Y \) is closer to 0.3 (e.g., Iben and Faulkner 1968; Demers and Harris 1974; Carson et al. 1981), a value of \( Y = 0.2 \) is taken to be consistent with current theory. Iben (1974, section 3.9), after reviewing five methods of estimating the mean helium abundance, has concluded that the helium abundance for globular cluster stars is "possibly not inconsistent with the value \( Y \sim 0.23 \) given by the simplest versions of big bang elements synthesis." Moreover, calculations with the equation of the blue edge itself, which had originally been constructed to yield new estimates for \( Y \), indicate a helium abundance of about 0.22 (Tuggle and Iben 1972).

For completeness, nevertheless, another one of Gingold's models was chosen, this time with the constraint that \( Y = 0.3 \). This model has a mass of 0.527 solar mass, a rate of change equal to \( 5.2 \times 10^{-7} \) cycle per year, and was also crossing the instability strip for the third time. However, the period of V714 Cyg (about 1.9 days) does not lie within the 3.4 - 5.2 day periods corresponding to the edges of the strip for this model.

It should be noted that the above results contrast with previous conclusions. For Gingold's models with \( Y = 0.3 \), Wehlau and Bohlender (1982) and Christianson (1983) found that some crossings gave rates of change of period of the correct order of magnitude for other BL Her stars. However, Wehlau and Bohlender apparently did not determine whether the ranges of period along the instability strip for those same crossings included the observed periods. Christianson concluded that her star, which has a period similar to that of V714 Cyg, is best
described by a model different from the one chosen by the procedure in
the present paper; apparently, she used a method different from the one
described above to find the average period of each crossing. Provencal
(1986) used a period-luminosity relationship to determine a theoretical
mass for her star and may have lost the resolution needed to
distinguish among Gingold’s models.

Lastly, because V714 Cyg lies far above the galactic plane, one
would expect the star to have a very small metal abundance; indeed, the
star was classified as Population II by this criterion in the first
place. However, Harris (1985b) has noted that solar metal abundances
among field BL Her stars are not unusual. If V714 Cyg and similar
field stars are found to have a high metal content, one may question
whether they should even be compared with Gingold’s models, each of
which were computed with Z = 0.001.

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Figure 1. O - C diagram of V714 Cygni for the years 1941 to 1988, calculated from the elements JD 2432515.4872 + 1.8873679E. The parabola represents the least-squares solution. The bars indicate the greatest possible uncertainty in determined times of maximum. Filled circles are points computed by the author from MMO plates; open circles represent the data Miller collected from Harvard and Vatican plates.