A Practical Approach to Transforming Magnitudes onto a Standard Photometric System

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Abstract We describe a practical implementation, convenient for amateur use, of a method of transforming instrumental magnitudes onto a standard photometric system in differential CCD photometry.

1. Introduction

Most amateur astronomers using a CCD camera to measure the brightness of an object of interest, be it a variable star, an asteroid, or some other distant light source, employ a procedure called differential photometry. In this the magnitude of the target object is found by comparing its brightness to other stars in the same field of view whose magnitudes are known. The measure of an object's brightness found by integrating the image of the object recorded by the CCD is called its instrumental magnitude. Increasingly amateurs are being encouraged to observe using photometric filters and to transform their measured instrumental magnitudes onto one of the standard photometric magnitude systems.

Transforming instrumental magnitudes in this way greatly increases their usefulness for scientific analysis. They can be combined with similarly-transformed measurements from other observers into a single internally-consistent dataset. The most common standard photometric system in use by amateurs today is the Johnson-Cousins UBVRI system and it is this which we shall use here, in particular its BVRI subset. However, the approach described is applicable to any standard system.

There are two main reasons why instrumental magnitudes differ from standard magnitudes: atmospheric extinction and a mismatch between the response of the equipment in use and the standard system. By observing stars whose magnitudes are very accurately known ("standard" stars), corrections can be found which enable instrumental magnitudes to be transformed onto the standard system. Over the years several formulations of these transformations have been devised and published, usually with the expectation that photometric skies will be readily available for their use. We describe here a variation on previously published methods which amateurs may find better suited to conditions they are likely to experience.

First we explain how atmospheric extinction and instrumental characteristics affect observations and briefly discuss the scope of applicability of the proposed approach. Next we describe the transformation equations and discuss choosing standard stars. We then explain how to find the transformation parameters and use them to transform instrumental magnitudes and color indices. Finally we use measurements of Landolt standard fields to illustrate the benefits of transforming instrumental magnitudes.

2. Atmospheric extinction and instrumental transformation

Atmospheric extinction has two principal causes: scattering of incident light by aerosols including dust and water vapor, and molecular Rayleigh scattering. Aerosol scattering is relatively uniform across the spectrum, rising slowly at shorter wavelengths, and is the dominant cause of extinction at wavelengths longer than about 500 nm. Rayleigh scattering increases very rapidly at shorter wavelengths and dominates at the blue end of the spectrum. The amount of aerosol scattering can vary widely depending on the aerosol content of the atmosphere, whereas Rayleigh scattering remains relatively constant. Further information about atmospheric extinction can be found in (Green 1992) and (Stubbs *et al.* 2007).

Wavelength-independent scattering is represented by a first order extinction coefficient multiplied by the air mass in the direction of the star, and wavelength-dependent scattering by the product of a second order extinction coefficient, the air mass, and the color index of the star. Since first order extinction is primarily caused by aerosol scattering it can potentially change rapidly during a single night or from night to night as the dust or moisture content of the atmosphere changes. Second order extinction is mainly due to Rayleigh scattering and is generally considered to be stable over time at a given location. Because of the rapid rise in Rayleigh scattering at the blue end of the spectrum, second order extinction should primarily affect measurements made through a B filter while measurements through V, R, and I filters will be much less affected.

The presence of a thin layer of cloud or haze reduces the incoming light equally at all wavelengths, as verified experimentally (Honeycutt 1971). In practice this adds a constant "gray" term to atmospheric extinction, which has the effect of changing the image zero point. Thin cloud is often difficult to detect visually but its presence need not prevent good quality photometry provided it is uniform and stable (Hardie 1959). Nevertheless, the most reliable results will always be obtained under clear skies.

In practice all BVRI filter and CCD detector combinations differ in their spectral response from the standard Johnson-Cousins system. This means that they will produce results which differ from the standard system and this difference will vary with the color of the star. Since the mismatch is usually small, it is normally assumed that it can be corrected by a linear term comprising the product of an instrumental transformation coefficient and the color index of the star. Provided the instrumental components do not change, any change in the transformation coefficient, for instance due to long term changes in the transmission properties of the optical components and filters, is likely to be small and slow.

So, while first order extinction may change from night to night, and even during a single night as atmospheric conditions change, any changes in second order extinction at the same location and instrumental transformation with the same equipment are likely to happen only slowly over time as noted in Welch (1979) and Harris *et al.* (1981). It would nevertheless be prudent to remeasure them at regular intervals to monitor for any such change.

3. Applicability

The approach to obtaining transformation parameters described here is aimed at those using CCD cameras to image the sky through sufficiently long focal length optical equipment that the field of view is small, typically less than one degree across. In photometry it is normally good practice to work above an altitude of about 30 degrees, corresponding to an air mass of 2, to avoid the worst effects of atmospheric extinction. Under these circumstances we can assume that all stars being measured in the same image are at the same zenith distance and hence are measured through the same air mass. The error introduced by this assumption is, in the worst case, only a few thousandths of a magnitude. It should be noted however that under some circumstances, for example when imaging large fields using DSLR cameras or imaging close to the horizon where air mass increases rapidly with reducing altitude, it may not be valid to assume that all stars in the image have the same air mass and a more rigorous analysis must be carried out.

4. The transformation equations

We take as our starting point the following equations which are conventionally used to transform instrumental magnitudes (in lower case) into magnitudes in a standard photometric system (in upper case), see for example Da Costa (1992) and Warner (2009).

$$B = b - k'_b X - k''_{bbv} X(B-V) + T_{bbv}(B-V) + Z_b \quad (1)$$

$$V = v - k'_v X - k''_{vbv} X(B-V) + T_{vbv}(B-V) + Z_v \quad (2)$$

where B and V are the standard B- and V-band magnitudes of the star, b and v are the instrumental magnitudes of the star measured in B and V filters, k'_b and k'_v are the B- and V-band first order extinction coefficients, k''_{bbv} and k''_{vbv} are the B- and V-band second order extinction coefficients for the (B–V) color index, X is the air mass of the star, T_{bbv} and T_{vbv} are the B- and V-band instrumental transformation coefficients for the (B–V) color index, and Z_b and Z_v are the

B- and V-band image zero points which are of course the same for all stars in an image.

We adopt the convention that the first, or only, letter in a subscript indicates the filter passband, while the second and third letters indicate the relevant color index.

As we noted earlier, rapid atmospheric changes may cause first order extinction coefficients and image zero points to vary from image to image, whereas second order extinction and instrumental transformation coefficients should remain constant or change only slowly over time.

The first order extinction, second order extinction, and instrumental transformation coefficients have usually been determined by separate processes. The two common methods of finding the extinction coefficients are the Bouguer and Hardie methods. The Bouguer method involves following a close group of standard stars as they move through a wide range of air mass over the course of a night. The Hardie method requires closely timed observations of pairs of red and blue standard stars at widely separated air masses. Both methods depend on atmospheric extinction remaining constant over the wide ranges of time and/or the sky involved in making these measurements. Instrumental transformation coefficients are found by observing a closely spaced group of stars with a wide range of colors at low air mass. Descriptions of these methods can be found in, for example Welch (1979) and Romanishin (2002).

However, as pointed out in (Harris *et al.* 1981), determining these parameters separately in this way is sub-optimal and usually requires iteration to obtain the best solution. They argue that it is better to make use of all observations together to determine the required parameters. Our approach is a simplified version of that described by Harris *et al.* which meets the needs of differential CCD photometry and may be easier for amateurs to implement in practice. It has the merit of reaching a solution in stages with graphical verification at each stage rather than a "black-box" multilinear least squares approach. This makes it easier to spot problems with individual stars, internal inconsistencies within the data, or human error.

We can rearrange equations (1) and (2) as follows:

$$(B–b) = T_{bbv}(B–V) – k''_{bbv}X(B–V) + Z_b – k'_bX \quad (3)$$

$$(V–v) = T_{vbv}(B–V) – k''_{vbv}X(B–V) + Z_v – k'_vX \quad (4)$$

and then rewrite them as:

$$(B–b) = C_{bbv}(B–V) + Z'_b \quad (5)$$

$$(V–v) = C_{vbv}(B–V) + Z'_v \quad (6)$$

where $C_{bbv} = T_{bbv} – k''_{bbv}X$ incorporates the B-band instrumental transformation and second order extinction correction for the (B–V) color index, $C_{vbv} = T_{vbv} – k''_{vbv}X$ is a similar term for the V-band, $Z'_b = Z_b – k'_bX$ incorporates the B-band image zero point and first order extinction correction, and $Z'_v = Z_v - k'_vX$ is a similar term for the V-band.

Since we are working with small fields and can assume all stars in the image being measured have the same value of air mass X, the terms C_{bbv}, C_{vbv}, Z'_b, and Z'_v are the same for all stars in the image. If we plot (B–b) against (B–V) for all stars in an image, the gradient will give us C_{bbv} at the value of X for that image, and similarly plotting (V–v) against (B–V) gives C_{vbv}.

Similar equations to (5) and (6) relate magnitudes measured using other filters and color indices. For example:

$$(R-r) = C_{rvr}(V-R) + Z'_r \tag{7}$$

$$(I-i) = C_{ivi}(V-I) + Z'_i \tag{8}$$

where definitions of the terms C_{rvr}, C_{ivi}, Z'_r, and Z'_i follow by analogy with those above.

Hence, if we can measure values of C_{bbv}, C_{vbv}, C_{rvr}, and C_{ivi} (henceforth described collectively as the C parameters where, generically, $C = T - k''X$), each with the corresponding value of X, then we can determine the instrumental transformations T and second order extinction coefficients k''. As we shall see, these are the parameters required to transform differential CCD photometry.

5. Sources of standard stellar magnitudes

In order to measure these parameters, we need to identify groups of stars which (a) will fit within a small CCD field of view, (b) contain as many stars as possible with accurately known magnitudes in each of the filter passbands, and (c) span as large a range of color index as possible.

Traditionally, the gold standard for calibration is the set of equatorial standard stars measured over many years by Arlo Landolt, see Landolt (1992, 2009, 2011). These have a root mean square (rms) V magnitude uncertainty of about 0.004 mag. While Landolt standard stars have the advantage of being visible from all latitudes, for observers far from the terrestrial equator they do not rise very high in the sky and so traverse a relatively small range of air mass. Using only those standard stars would necessitate extrapolating to air mass values outside this range, a procedure which would inevitably reduce accuracy. Using only Landolt stars therefore limits the accuracy attainable when trying to account for air mass dependent effects. Another slight drawback of Landolt stars is that it is sometimes difficult to include many of them with a wide range of color index within a typical CCD field.

For this reason we investigated using groups of secondary standard stars which have themselves been calibrated using Landolt stars, in particular ones which culminate near the zenith at higher latitudes and can therefore be observed at or near an air mass of 1. Suitable fields of stars have been measured in the B, V, R, and I passbands by Arne Henden as an aid to selecting comparison stars for variables and are available from Henden (2012). When selecting stars from these fields to use as secondary standards, we excluded those which had fewer than ten observations, large measurement uncertainties, or close companions which might compromise their measurement. Also, stars significantly out of line in magnitude-color plots with the majority of stars in the same field were excluded on the grounds that their magnitudes may be incorrect or they might be variable. The ten brightest stars in each field satisfying these selection criteria have been used. The magnitude uncertainties of these stars are larger than for Landolt standard stars, but this is in part compensated by having more stars per field with a wider range of color. The stars used in the example given here were from fields around the variables EE Cep, TT Cas, V504 Per, and Var Cas 06 (also known as VSX J000921.8+543943—the bright gravitational microlens variable).

Figure 1 shows stars in the field of Var Cas 06. The circled stars were used as secondary standards and have V magnitudes in the range 11.373–13.889 with rms uncertainty 0.024 and B–V color indices in the range 0.125–1.716.

The availability of accurately measured stellar magnitudes across much of the sky from photometric surveys such as the AAVSO APASS survey (AAVSO 2012) should greatly increase the availability of high quality secondary standard stars suitable for this purpose.

6. Finding the instrumental transformation and second order extinction coefficients

First we select a suitable field containing stars with known, accurately-measured B, V, R, and I magnitudes covering as wide a range of colors as possible, such as the one shown in Figure 1. We identify the stars in the field we intend to use as our secondary standards. We then wait for a clear night with stable sky conditions when the field is well placed and take between five and ten images of the field through each filter. We take care to record the brightest stars with as high signal to noise as possible in each filter while avoiding non-linearity or saturation of the CCD. This typically takes only a few minutes for each filter during which we require the sky conditions to remain clear. We calibrate the images using dark and flat fields and using aperture photometry measure the instrumental magnitudes b, v, r, and i for each of our standard stars in each image taken with the corresponding filter. We then calculate mean values and standard deviations of b, v, r, and i for each star over all the images taken with each filter. We also calculate a mean value of X of all the stars in all the images taken with each filter.

To check that sky conditions have indeed remained stable throughout each set of images, we can compare standard deviations of the instrumental magnitudes of each star in each filter with the estimated measurement uncertainties output by the photometry software. If the former are substantially larger than the latter, it is an indication that sky conditions were probably unstable and the results should not be used for calibration purposes.

We know values of (B–V), (V–R), and (V–I) and can calculate values of (B–b), (V–v), (R–r), and (I–i) for each standard star. We can also calculate the uncertainties on (B–b), and so on, for each star by adding the quoted uncertainty on its standard magnitude B and the standard deviation of its instrumental magnitude b in quadrature.

We plot (B–b) vs (B–V), (V–v) vs (B–V), (R–r) vs (V–R), and (I–i) vs (V– I). Assuming the transformations we are seeking are linear, we expect the data points to lie approximately on a straight line. If the line is horizontal, it indicates that our measured instrumental magnitude has no color dependency. If, as is more likely, the line is at an angle to the horizontal it means that there is a color dependency in our observations which we need to correct.

Figure 2 shows magnitude-color plots of (B–b) vs (B–V), (V-v) vs (B–V), (R–r) vs (V–R), and (I–i) vs (V–I) for stars in the field of the variable EE Cep. The equipment used was a 0-35m SCT, SXVR-H9 CCD camera, and Astrodon dichroic B, V, R, and I filters.

From equations (5) to (8) the gradients of straight lines fitted to the data points in these plots give values of the C parameters at the mean air mass X calculated for each filter, and we can calculate their uncertainties from the scatter in the data points about the fitted lines.

We repeat this process for several more fields of secondary standard stars satisfying our selection criteria and covering as wide a range of air mass as possible. Each field provides values of the C parameters and a corresponding value of X for each filter. The data do not all have to be collected on the same night since, as noted earlier, these parameters should be stable over time so data from several nights can be combined.

Since for each of the C parameters, $C = T - k''X$, if we now plot the value of the C parameter for each filter against the corresponding value of X and make weighted linear fits, we can obtain values for the instrumental transformation (T) and second order extinction (k") coefficients for each filter with estimates of their uncertainties. In these fits we weight each value of C with the inverse square of its uncertainty.

Figure 3 shows plots of the C parameters vs X obtained from twelve sets of observations of four fields imaged over two nights. Also shown in these plots are the straight lines representing the fits for T and k".

Table 1 lists the values of the instrumental transformation and second order extinction coefficients found from these analyses. Remember our convention is that the first letter in a subscript indicates the filter passband, while the second and third letters indicate the relevant color index.

The second order extinction coefficients for the V, R, and I filter passbands are small and consistent with zero within experimental uncertainty. This is as expected given that extinction in these filters is primarily due to aerosol scattering which has minimal wavelength dependency. Therefore, consistent with conventional wisdom, we will assume these have a value of zero. The second order extinction coefficient for the B filter is significantly non-zero as expected from the rapid rise in Rayleigh scattering at shorter wavelengths.

By observing several fields of secondary standard stars covering a wide range of color and air mass, slight differences between the assumed "standard" magnitudes for individual stars and their "true" values are averaged out. In this way we can offset the lower accuracy in the magnitudes of our secondary standard stars compared to primary standards.

7. First order extinction coefficients

Values of the first order extinction coefficients k' are not calculated directly and are not needed for differential CCD photometry with small fields where we assume X is constant over the field. They are contained in the image zero points $Z' = Z - k'X$ in eqns (5) to (8). Values of Z' can be found from intercepts of the linear fits in Figure 2 at color index 0 but in general these will not yield consistent values of Z and k' since these parameters vary with changes in atmospheric transparency. Nevertheless, if conditions are sufficiently stable during one night, we might expect Z and k' to remain approximately constant. In that case, if we plot the values of Z' for each filter against the corresponding values of X for data obtained on that night, we should get a straight line whose gradient gives the mean value of k' for the appropriate filter on that night (see Figure 4). The corresponding values of k' are listed in Table 2. As expected, k' gradually diminishes as the wavelength increases, reflecting the reduction in aerosol scattering at longer wavelengths.

8. Transforming instrumental magnitudes to standard magnitudes

Now we have all the parameters needed to transform instrumental magnitudes measured in differential CCD photometry onto the BVRI standard magnitude system. In doing this it is easier to work with equations which contain only instrumental rather than standard magnitudes on the right hand side. By simple manipulation of eqns (5) to (8) we derive the equations we require.

$$B = b + C'_{bbv}\,(b-v) + z_b \tag{9}$$

$$V = v + C'_{vbv}\,(b-v) + z_v \tag{10}$$

$$R = r + C'_{rvr}\,(v-r) + z_r \tag{11}$$

$$I = i + C'_{ivi}\,(v-i) + z_i \tag{12}$$

where the C' parameters are related to the original C parameters as follows:

$$C'_{bbv} = C_{bbv} / (1 - C_{bbv} + C_{vbv}), \quad (13)$$

$$C'_{vbv} = C_{vbv} / (1 - C_{bbv} + C_{vbv}), \quad (14)$$

$$C'_{rvr} = C_{rvr} / (1 - C_{vvr} + C_{rvr}), \quad (15)$$

$$C'_{ivi} = C_{ivi} / (1 - C_{vvi} + C_{ivi}), \quad (16)$$

and the various "z" terms on the right hand side are image zero points which are the same for all stars in an image.

Since, generically, $C = T - k''X$ and the values of T and k'' are known (see Table 1), if we know the air mass X of an image we can calculate the values of the appropriate C parameters and hence the C' parameters.

Suppose we want to find the standard V magnitude of a variable star in a field containing several comparison stars with known magnitudes. We take several images of the field through B and V filters and measure the instrumental magnitudes b and v of the variable and comparison stars in each image. Knowing the mean air mass X of the stars in each image we calculate $C_{bbv} = T_{bbv} - k''_{bbv}X$ and $C_{vbv} = T_{vbv} - k''_{vbv}X$ and hence C'_{vbv} from equation (14) for each image. Using equation (10) and knowing the standard and instrumental magnitudes for the comparison stars, we can determine the zero point z_v for each image. Since we know the instrumental b and v magnitudes of the variable, we can again use equation (10) to calculate its standard V magnitude in each image. Finally, using these individual measurements of the V magnitude of the variable, we can compute its mean and standard deviation.

A similar procedure will yield values for B, R, and I calculated from the measured instrumental magnitudes b, v, r, and i using eqns (9), (11), and (12).

9. Transforming color indices

The transformation equations for color indices can be found from equations (9) to (12).

$$(B-V) = C'_{bv} (b-v) + z_{bv} \quad (17)$$

$$(V-R) = C'_{vr} (v-r) + z_{vr} \quad (18)$$

$$(V-I) = C'_{vi} (v-i) + z_{vi} \quad (19)$$

where

$$C'_{bv} = 1 / (1 - C_{bbv} + C_{vbv}), \quad (20)$$

$$C'_{vr} = 1 / (1 - C_{vvr} + C_{rvr}), \quad (21)$$

$$C'_{vi} = 1 / (1 - C_{vvi} + C_{ivi}), \quad (22)$$

$$z_{bv} = (z_b - z_v) \quad (23)$$

$$z_{vr} = (z_v - z_r) \quad (24)$$

and

$$z_{vi} = (z_v - z_i). \quad (25)$$

Here the two subscript letters indicate the relevant color index.

10. Implementation

In practice, the two procedures described above—finding the transformation parameters and using them to transform instrumental magnitudes to standard magnitudes—are quite straightforward to implement in a spreadsheet.

11. Transforming magnitudes measured for Landolt standard fields

As an application of this approach, BVR-filtered instrumental magnitudes were measured for stars in three Landolt standard fields and transformed as described above. The rms residuals between the standard and derived magnitudes before and after transformation are shown in Table 3. These results clearly show the improvement achieved by transforming magnitudes onto the standard system.

12. Conclusion

We have described and demonstrated a practical approach to finding and applying the transformations required to bring instrumental magnitudes onto a standard photometric system in differential CCD photometry. This includes correcting for second order atmospheric extinction where appropriate. It is possible to use well-measured secondary standard stars in fields spanning a wide range of declinations to enable full coverage of the air mass range from observing sites far from the terrestrial equator. Sky conditions must remain clear and stable for long enough to obtain short series of filtered images of the fields required to cover the required range of air mass. These images can be obtained on several nights and the results combined. This approach may be easier for some observers, particularly those operating at high terrestrial latitudes, to use than other approaches.

13. Acknowledgements

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References

AAVSO. 2012, APASS: The AAVSO Photometric All-Sky Survey (http://www.aavso.org/apass), AAVSO, Cambridge, MA.

Da Costa, G. S. 1992, in *Astronomical CCD Observing and Reduction Techniques*, ed. S. B. Howell, Astron. Soc. Pacific, San Francisco, 90.

Green, D. W. E. 1992, *Int. Comet Q.*, **14**, 55.

Hardie, R. 1959, *Astrophys. J.*, **130**, 663.

Harris, W. E., Fitzgerald, M. P., and Reed, B. C. 1981, *Publ. Astron. Soc. Pacific*, **93**, 507.

Henden, A. H. 2012, "Calibration fields" (ftp://ftp.aavso.org/public/calib/).

Honeycutt, R. K. 1971, *Publ. Astron. Soc. Pacific*, **83**, 502.

Landolt, A. 1992, *Astron. J.*, **104**, 340.

Landolt, A. 2009, *Astron. J.*, **137**, 4186.

Landolt, A. 2011, "Equatorial standards" (http://www.cfht.hawaii.edu/ObsInfo/Standards/Landolt/).

Romanishin, W. 2002, "An Introduction to Astronomical Photometry Using CCDs" (http://homepage.usask.ca/~ges125/Astronomy/wrccd4a.pdf).

Stubbs, C. W., *et al.* 2007, *Publ. Astron. Soc. Pacific*, **119**, 1163.

Warner, B. 2009, Photometry Workshop, Soc. Astron. Sciences, 28th Annual Symposium on Telescope Science, held May 19–21, 2009, Big Bear Lake, CA.

Welch, D. 1979, *J. Roy. Astron. Soc. Canada*, **73**, 370.

Table 1. Instrumental transformation and second order extinction coefficients.

Transformation Coefficients		*Second Order Extinction Coefficients*	
T_{bbv}	0.0295 ± 0.0104	k''_{bbv}	–0.0199 ± 0.0067
T_{vbv}	–0.0335 ± 0.0091	k''_{vbv}	0.0016 ± 0.0055
T_{rvr}	–0.1262 ± 0.0164	k''_{rvr}	–0.0077 ± 0.0102
T_{ivi}	–0.0642 ± 0.0165	k''_{ivi}	0.0019 ± 0.0095

Table 2. First order extinction coefficients for data obtained on one night under stable conditions.

First Order Extinction Coefficients	
k'_b	0.376 ± 0.015
k'_v	0.245 ± 0.016
k'_r	0.224 ± 0.025
k'_i	0.166 ± 0.029

Table 3. rms residuals between standard and derived B, V, and R magnitudes before and after transformation for stars in three Landolt standard fields.

Field	*Air mass*	*rms residuals between standard and derived magnitudes*					
		untransformed			*transformed*		
		B	*V*	*R*	*B*	*V*	*R*
98-185	1.63	0.064	0.028	0.033	0.021	0.008	0.008
98-618	1.64	0.035	0.026	0.055	0.015	0.007	0.008
114-750	1.57	0.031	0.018	0.031	0.009	0.007	0.009
Mean	—	0.043	0.024	0.040	0.015	0.007	0.008

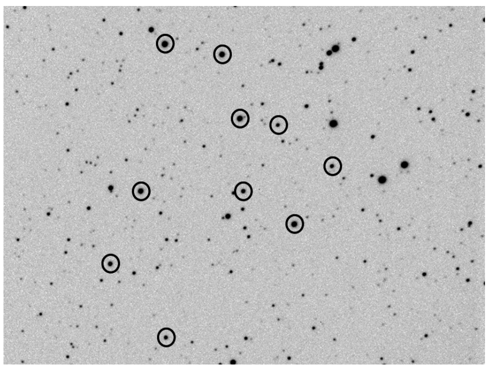

Figure 1. Stars in the field of Var Cas 06 used as secondary standards.

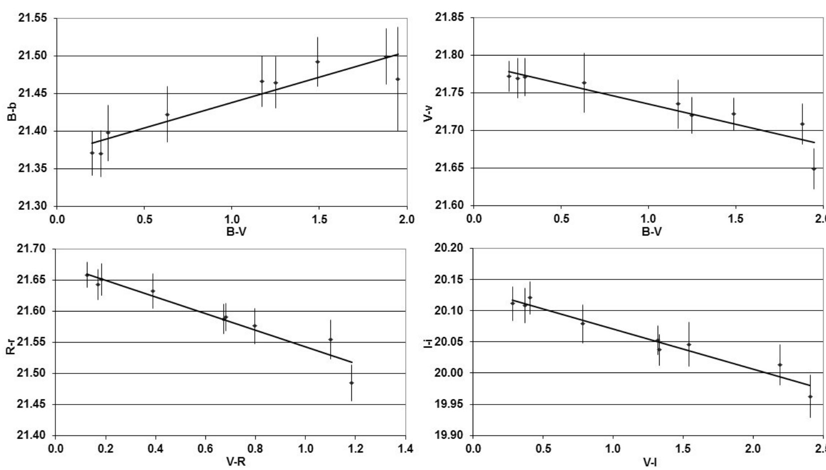

Figure 2. (B–b) vs (B–V), (V–v) vs (B–V), (R–r) vs (V–R) and (I–i) vs (V–I) for stars measured in the field of the variable EE Cep.

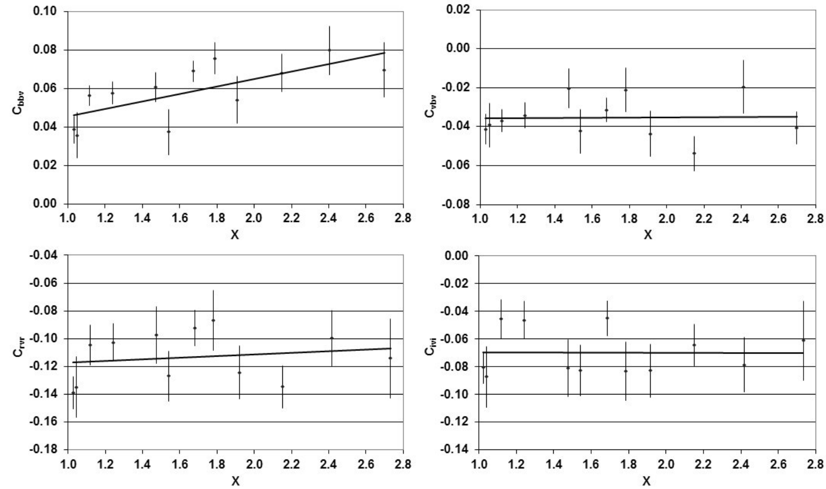

Figure 3. C_{bbv}, C_{vbv}, C_{rvr}, and C_{ivi} vs X for 12 sets of observations of four fields imaged over two nights.

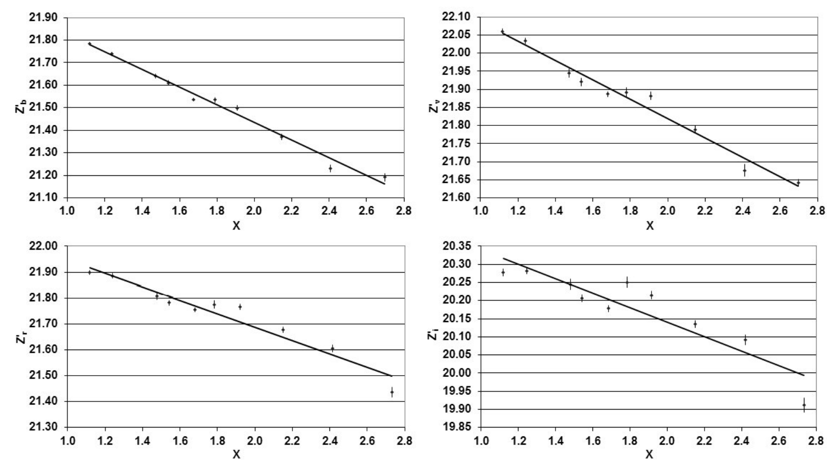

Figure 4. Z'_b, Z'_v, Z'_r, and Z'_i vs X for the data from one night under stable conditions.