CCD Photometry and Roche Modeling of the Eclipsing Overcontact Binary Star System TYC 01963-0488-1

Kevin B. Alton
UnderOak Observatory, 70 Summit Avenue, Cedar Knolls, NJ 07927; kbalton@optonline.net

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Abstract  TYC 01963-0488-1 (ASAS J094440+2632.1) is a W UMa binary system (P = 0.427036 d) which has been largely overlooked since first being detected nearly 15 years ago by the ROTSE-I telescope. Other than the monochromatic ROTSE-I survey data, no multi-colored light curves have been published. Photometric data collected in three bandpasses (B, V, and I) at UnderOak Observatory produced five new times-of-minimum for TYC 01963-0488-1 which were used to establish a linear ephemeris from the first Min I epoch (HJD0). No published radial velocity data are available for this system; however, since this W UMa binary undergoes very obvious total eclipses, Roche modeling yielded a well-constrained photometric value for q (~0.25). There is a suggestion from the ROTSE-I data and new results herein that Max II is more variable than Max I. Therefore, Roche model fits for the TYC 01963-0488-1 light curves collected in 2015 were assessed with and without spots.

1. Introduction

2. Observations and data reduction

2.1. Photometry

The variable behavior of TYC 01963-0488-1 was first observed during the ROTSE-I CCD survey (Gettel et al. 2006); the system was later classified by Hoffman et al. (2009). Photometric data are accessible on the Northern Sky Variable Survey website (Wozniak et al. 2004). Other than the reduced on-line data (Gettel et al. 2006) at VizieR and an entry in the International Variable Star Index (VSX; Watson et al. 2014), no other reference to this binary system was found in the literature. The paper herein marks the first robust determination of orbital period and Roche model assessment of TYC 01963-0488-1 which has been published.

2.2. Light curve analyses

2.2.1. Photometry and ephemerides

Five stars in the same FOV with TYC 01963-0488-1 which were used to derive catalog-based (MPOSC3) magnitudes (Table 1) showed no evidence of inherent variability over the period of image acquisition and stayed within ±0.03 magnitude for V and I filters and ±0.05 for B. Photometric values in B (n = 407), V (n = 407), and I (n = 410) were processed to generate three LCs that spanned 45 days between March 24 and May 8, 2015 (Figure 1). In total, four new secondary (s) and one primary (p) minima were captured during this investigation; data from all filters were averaged for each session (Table 2) since no color dependency on the timings was noted. A period determination (P = 0.427028 ± 0.000008) from unfiltered data (ROTSE-I) collected in 1999–2000 was made using PERANSO (Vannmunster 2006) by employing periodic orthogonal fits (Schwarzenberg-Czerny 1996) to fit observations and analysis of variance (ANOVA) to evaluate fit quality. After converting magnitude to flux, ROTSE-I and Underoak Observatory (UO) light curve data (V mag.) were then folded together; the best fit was found where the orbital period was 0.427036 ± 0.000004 day (Figure 2). The Fourier routine (FALC) in MPO CANOPUS provided a similar period solution (0.427036 ± 0.000006) using only the multicolor data from UO. The first epoch (HJD0) for this eclipsing binary is therefore defined by the following linear ephemeris equation:

Min. I hel. = 2457150.63657 (3) + 0.427036 (4) E (1)
Table 1. Astrometric coordinates (J2000) and MPOSC3 catalog magnitudes (B, V, and I<sub>c</sub>) for TYC 01963-0488-1 and five comparison stars used in this photometric study.

<table>
<thead>
<tr>
<th>Star Identification</th>
<th>R.A.</th>
<th>Dec.</th>
<th>MPOSC3 B</th>
<th>MPOSC3 V</th>
<th>MPOSC3 I&lt;sub&gt;c&lt;/sub&gt;</th>
<th>MPOSC3 (B-V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYC 01963-0488-1</td>
<td>09 44 40.44</td>
<td>26 32 07.2</td>
<td>11.57–12.11&lt;sup&gt;b&lt;/sup&gt;</td>
<td>11.17–11.63&lt;sup&gt;b&lt;/sup&gt;</td>
<td>10.69–11.15&lt;sup&gt;o&lt;/sup&gt;</td>
<td>0.399</td>
</tr>
<tr>
<td>TYC 01963-0389-1</td>
<td>09 44 44.82</td>
<td>26 39 33.5</td>
<td>12.574</td>
<td>11.830</td>
<td>11.017</td>
<td>0.744</td>
</tr>
<tr>
<td>GSC 01963-00146</td>
<td>09 44 52.06</td>
<td>26 42 27.5</td>
<td>13.288</td>
<td>12.685</td>
<td>12.000</td>
<td>0.603</td>
</tr>
<tr>
<td>TYC 01963-0461-1</td>
<td>09 44 07.14</td>
<td>26 36 36.9</td>
<td>12.272</td>
<td>11.418</td>
<td>10.508</td>
<td>0.854</td>
</tr>
<tr>
<td>GSC 01963-00102</td>
<td>09 44 46.83</td>
<td>26 33 28.5</td>
<td>14.237</td>
<td>13.722</td>
<td>13.120</td>
<td>0.515</td>
</tr>
<tr>
<td>GSC 01963-00586</td>
<td>09 44 44.41</td>
<td>26 34 30.3</td>
<td>14.501</td>
<td>13.971</td>
<td>13.355</td>
<td>0.530</td>
</tr>
</tbody>
</table>

<sup>a</sup>: MPOSC3 is a hybrid catalog which includes a large subset of the Carlsberg Meridian Catalog (CMC-14) as well as from the Sloan Digital Sky Survey (SDSS).

<sup>b</sup>: Range of observed magnitudes in UO light curves for TYC 01963-0488-1.

Table 2. New times-of-minimum for TYC 01963-0488-1 acquired at UnderOak Observatory.

<table>
<thead>
<tr>
<th>Mean Observed Time-of-Minimum (HJD–2400000)</th>
<th>± Error</th>
<th>UT Date of Observations</th>
<th>Type of Minimum&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>57114.5523</td>
<td>0.0002</td>
<td>02Apr2015</td>
<td>s</td>
</tr>
<tr>
<td>57117.5433</td>
<td>0.0001</td>
<td>05Apr2015</td>
<td>s</td>
</tr>
<tr>
<td>57134.6238</td>
<td>0.0002</td>
<td>22Apr2015</td>
<td>s</td>
</tr>
<tr>
<td>57137.6136</td>
<td>0.0001</td>
<td>25Apr2015</td>
<td>s</td>
</tr>
<tr>
<td>57150.6366</td>
<td>0.0002</td>
<td>08May2015</td>
<td>p</td>
</tr>
</tbody>
</table>

<sup>a</sup>: s = secondary; p = primary.

Table 3. Difference in light curve minima and maxima by bandpass.

<table>
<thead>
<tr>
<th>Band</th>
<th>Max II – Min I</th>
<th>Min I – Min II</th>
<th>Min I – Max I</th>
<th>Min II – Max II</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.015</td>
<td>0.0739</td>
<td>0.5092</td>
<td>0.4113</td>
</tr>
<tr>
<td>V</td>
<td>0.005</td>
<td>0.0581</td>
<td>0.4766</td>
<td>0.4131</td>
</tr>
<tr>
<td>I&lt;sub&gt;c&lt;/sub&gt;</td>
<td>0.004</td>
<td>0.0405</td>
<td>0.4551</td>
<td>0.4110</td>
</tr>
</tbody>
</table>

Figure 1. Folded CCD light curves for TYC 01963-0488-1 produced from photometric data obtained between March 24 and May 8, 2015. The top (I<sub>c</sub>), middle (V), and bottom curve (B) shown above were reduced to MPOSC3-based catalog magnitudes using MPO CANOPUS.

Figure 2. Survey data from the ROTSE-I telescope and photometric results (V mag.) collected at UnderOak Observatory were folded together using period analysis (P = 0.427036 ± 0.000004 d). Greater scatter at Min II (φ = 0.75) suggests the possibility of an active photosphere for TYC 01963-0488-1.

Figure 3. Color change (B–V) of TYC 01963-0488-1 during varying phases of eclipse was evaluated from binned data (φ = 0.002). Relatively minor deviations from the solid-line mean (0.423 ±0.014) suggest that the effective temperatures of both stars are not very different.
No attempt was made to determine whether there are secular changes in eclipse timings over time since these data were only collected over 45 days.

3.2. Light curve behavior

As expected from a overcontact binary system, LCs (Figure 1) exhibit minima which are separated by 0.5 phase (\(q\)) and consistent with a synchronous circular orbit. Flattened bottoms at Min I and Min II strongly suggest that this binary system undergoes total eclipses. Data from the ROTSE-I survey tend to exhibit greater variability around Max II compared to Max I (Figure 2). A slightly positive O’Connell effect (Max II fainter than Max I) is observed most notably with the B passband during the 2015 campaign whereas both V and I\(_{\gamma}\) passbands exhibit vanishingly smaller differences (Table 3). In general this effect has been attributed to the presence of cool starspot(s), hot region(s), gas stream impact on either or both of the binary cohorts, and/or other unknown phenomena which produce surface inhomogeneities (Yakut and Eggleton 2005). The net result can be unequal heights during maximum light and is often simulated by the introduction of starspots during Roche-type modeling of the LC data.

3.3. Spectral classification

Data from folded LCs (V- and B-mag) were binned into equal phase intervals (0.002) to produce a difference (B–V) plot (Figure 3). It would appear that changes in color with phase are small (0.423 ± 0.014) suggesting relatively minor differences in effective temperature between stars. Color index (B–V) data from UO and three other surveys (Table 4) were corrected using the reddening value (E(B–V) = 0.0175 ± 0.0005) observed within a 5 arcmin radius of TYC 01963-0488-1 (Schlafly and Finkbeiner 2011; Schlegel et al. 1998). The mean result, (B–V)\(_{\gamma}\) = 0.393 ± 0.014, which was adopted for subsequent Roche modeling, indicates that the most luminous star in this system has an effective temperature of 6705 K, and ranges between spectral type F2V and F3V (Pecaut and Mamajek 2013).

3.4. Roche modeling approach

In the absence of radial velocity (RV) data, it is not possible to unequivocally determine \(q\), the mass ratio (\(m_1 / m_2\)), or whether TYC 01963-0488-1 is an A- or W-type overcontact binary system. According to Binnendijk (1970), the deepest minimum (Min I) of an A-type W UMa variable results from the eclipse of the hotter more massive star by the cooler less massive cohort. In this context to W-type stars where the deepest minimum results from the hotter but less massive star being eclipsed by the more massive but cooler cohort. In general, A-type W UMa variables can be characterized by their total mass (\(M_1 > 1.8 M_2\)), spectral class (A–F), orbital period (\(P > 0.4\) d), extent of thermal contact (\(f\)), propensity to exhibit a total eclipse due to large size differences, mass ratio (\(q < 0.3\)), and the temperature difference (\(\Delta T < 100\) K) between the hottest and coolest star (Skleton and Smits 2009). On balance, when all of these factors are considered, TYC 01963-0488-1 would appear to best fit those parameters which define an A-subtype. A reliable photometric value for mass ratio (\(q_{\gamma}\)) can be determined but only for those W UMa systems where a total eclipse is observed from our vantage point (Terrell and Wilson 2005). A recent paper published by Hambálek and Pribulla (2013) offered an approach to estimate mass ratio (\(q\)) and the orbital inclination (\(i\)) prior to Roche modeling. These investigators employed the code ROCHE (Pribulla 2012) to simulate a total of 11,895 LCs from overcontact binaries as defined by varying values for three parameters (\(q\) (0.05–1;\(\Delta = 0.025\)), fill-out (\(f = 0.25, 0.5, 0.75, 1\)), and \(i\) (30°–90°;\(\Delta = 1°\))). A LC from an eclipsing binary can be represented by trigonometric polynomials such that the corresponding Fourier coefficients define the “informational contents” of the system. A partial eclipse can be adequately represented by a 10th order trigonometric polynomial, whereas a total eclipse is more difficult to model. In this case only the first 11 Fourier coefficients (\(a_n-a_1\)) were used to investigate the uniqueness of each solution (Table 5). The amplitude of the LC and the minima width are respectively constrained by the values for \(a_1\) and \(a_2\). The uniqueness (\(\delta q\)) for a photometric mass ratio (\(q_{\gamma}\)) solution is defined by the \(a_2-a_1\) plane (Hambálek and Pribulla 2013) and varies according to the number and precision of the LC data points. The authors have conveniently provided an on-line link to UNIQUE (http://www.ta3.sk/~lhamblek/download/unique.zip), which can be used to calculate the geometric elements [\(q\), \(f\), and \(i\)] along with the corresponding Fourier coefficients.

In preparation for analysis by UNIQUE, monochromatic LC data (\(n = 407\); V mag.) collected at UO were converted to normalized flux and then binned into constant phase intervals (0.002) to satisfy a requirement of the program. When third light (\(I\)) was assumed to be zero, the best match from the LC data library (coef.dat) used by UNIQUE corresponded to \(q = 0.225, f = 0.5,\) and \(i = 79°\). Roche modeling of LC data

Table 4. Spectral classification of TYC 01963-0488-1 based upon dereddened\(^a\) (B–V) data from various surveys and the present study.

<table>
<thead>
<tr>
<th>Stellar Attribute</th>
<th>MPOSC3</th>
<th>2MASS</th>
<th>SDSS-DR8</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B–V)(_{\gamma})</td>
<td>0.382</td>
<td>0.381</td>
<td>0.407</td>
<td>0.404</td>
</tr>
<tr>
<td>±0.094</td>
<td>±0.050</td>
<td>±0.04</td>
<td>±0.014</td>
<td></td>
</tr>
<tr>
<td>(T_{eff}) (K)</td>
<td>6765</td>
<td>6765</td>
<td>6656</td>
<td>6673</td>
</tr>
<tr>
<td>Spectral Class(^b)</td>
<td>F2-F3V</td>
<td>F2-F3V</td>
<td>F3-F4V</td>
<td>F3-F4V</td>
</tr>
</tbody>
</table>

\(^a\) E(B–V) = 0.0175 ± 0.0005. \(^b\) \(T_{eff}\), interpolated and spectral class assigned from Pecaut and Mamajek (2013). c: Mean value for (B–V)\(_{\gamma}\) = 0.393 ± 0.014; \(T_{eff}\) = 6705 K; Spectral type = F2 to F3V.
from TYC 01963-0488-1 was primarily accomplished using the program phoebe 0.31a (Prša and Zwitter 2005). The model selected was for an overcontact binary not in thermal contact (Mode 3) and each curve was weighted based upon observational scatter. Bolometric albedo ($A_{1,2} = 0.5$) and gravity darkening coefficients ($g_{1,2} = 0.32$) for cooler stars (< 7500 K) with convective envelopes were assigned by theory according to Ruciński (1969) and Lucy (1967), respectively. The effective temperature of the more massive primary star was fixed ($T_{\text{eff}1} = 6705$ K) according to the earlier designation as spectral type F2V to F3V. Following any change in the effective temperature for the secondary ($T_{\text{eff}2}$), new logarithmic limb darkening coefficients ($x_{1,2}, y_{1,2}$) were interpolated according to Van Hamme (1993). All parameters except for $T_{\text{eff}1}$, $A_{1,2}$, and $g_{1,2}$ were allowed to vary during DC iterations. Roche modeling was initially seeded with $q = 0.225$ and $i = 79^\circ$ from UNIQUE (Hambálek and Pribulla 2013) and a lower effective temperature for the secondary ($T_{\text{eff}2} = 6600$ K) in order to comply with the definition of an A-type system. This assessment included synthesis of light curves for TYC 01963-0488-1 with and without the incorporation of a cool spot to address the so-called O’Connell effect (Max I brighter than Max II). The possibility of hot spots which can also produce LC asymmetry cannot be discounted; however, the appearance of cold spot(s) can be persistent and could explain the same Max II asymmetry observed between 1999 and 2000 by the ROTSE-I survey (Akerlof et al. 2000).

3.5. Modeling results

3.5.1. Light curve analysis

The initial estimates for $q$, $i$, and $T_{\text{eff}2}$ quickly converged to best fit Roche model solutions. In this case the values ($q$ and $i$) derived from UNIQUE were reasonably close to the final values determined from Roche modeling (Table 6). Corresponding unspotted (Figure 4) and spotted (Figure 5) simulations reveal that the addition of a cool spot on the less massive secondary star resulted in a modestly improved fit ($\chi^2$) of these multi-color data. A pictorial model rendered with bm3 using the physical and geometric elements from the system with a cool spot on the secondary star is shown in Figure 6. After a best model fit was found, values and errors for $T_{\text{eff}2}$, $i$, $q$, and $\Omega_{1,2}$ were further examined using the phoebe scripter in which the wd minimization program (dc) was programatically executed 1,000 times (Bonanos 2009). During each heuristic scan, input parameter values were updated for the next iteration and the formal error derived from the standard deviations; a representative example (Figure 7) illustrates the probabilistic relationship between $q$, $i$, and $\Delta \chi^2$. The fairly steep boundary which defines the 99.99% confidence interval ($\Delta \chi^2 = 15.1$) is consistent with a well-constrained value for $q$ (0.248 ± 0.002).

The fill-out parameter ($f$), which is a measure of the shared photospheric volume between each star, was calculated according to Bradstreet (2005) as:

$$f = \frac{\left(\Omega_{\text{outer}} - \Omega_{1,2}\right)}{\left(\Omega_{\text{inner}} - \Omega_{\text{outer}}\right)}$$  \hspace{1cm} (2)
3.6. Absolute Parameter Estimates

Absolute parameters (Table 7) were derived for each star in this A-type W UMa binary system using results from the best fit simulation (spotted model) of the 2015 LC. In the absence of RV data (v, + v,), total mass can not be calculated directly. However, stellar mass and radii from binary systems have been tabulated over a wide range of spectral types (Harmanec 1988) where the primary star (Teff = 6705 K) in TYC 01963-0488-1 is estimated to have a mass of 1.48 ± 0.06 M⊙. Alternatively, two different empirical period-mass relationships for W UMa-binaries have been reported by Gazeas and Stepień (2008) and Qian (2003). According to Gazeas and Stepień (2008) the mass of the primary star (M₁) can be calculated from the expression (3):

$$\log M_1 = (0.755 \pm 0.059) \log P + (0.416 \pm 0.024),$$

while the mass of the secondary (M₂) can be similarly calculated from the orbital period (P) with the following relationship (4):

$$\log M_2 = (0.352 \pm 0.166) \log P - (0.262 \pm 0.067)$$

This leads to M₁ = 1.37 ± 0.1 M⊙ for the primary and M₂ = 0.405 ± 0.086 M⊙ for the secondary star. The value for the photometrically determined mass ratio (q, = 0.248 ± 0.002) from this study is contained within the mass ratio and error (q, = 0.296 ± 0.066) calculated using equations 3 and 4. Another mass-period relationship (equation 5) was derived by Qian (2003) and largely corresponds to A-type W UMa systems where M₁ > 1.35 M⊙ and P > 0.41 d.

$$\log M_1 = 0.761 (\pm 0.150) + 1.82 (\pm 0.28) \times P$$

In this case the solution leads to a somewhat higher estimate for the primary star mass (M₁ = 1.54 ± 0.20). The average of all three values (M₁ = 1.46 ± 0.07) was used for subsequent determinations of M₁, semi-major axis (a), volume-radius (r₁), bolometric magnitude (M₁), and distance (pc) to TYC 0163-0488-1. The semi-major axis, a(R₁) = 2.91 ± 0.04, was calculated according to Kepler’s third law where:

$$a = \frac{G \times P^2 (M_1 + M_2)}{(4\pi^2)}$$

According to the expression (7) derived by Eggleton (1983), the effective radius of each Roche lobe (r₁) can be calculated to within an error of 1% over the entire range of mass ratios (0 < q < ∞):

$$r_1 = \frac{0.49 q^{1/3} + \ln(1 + q^{1/3})}{0.6 q^{2/3}}$$
Volume-radius values were determined for the primary \((r_1 = 0.5021 \pm 0.0001)\) and secondary \((r_2 = 0.2670 \pm 0.0001)\) stars. The absolute radii for both binary constituents can then be calculated where \(R_1 = a \times r_1 = 1.46 \pm 0.02 R_\odot\) and \(R_2 = a \times r_2 = 0.78 \pm 0.01 R_\odot\). The bolometric magnitude \(M_{\text{bol1,2}}\) and luminosity \((L_{1,2})\) in solar units \((L_\odot)\) for the primary and secondary stars were calculated from well-established relationships where

\[
M_{\text{bol1,2}} = 4.75 - 5 \log \left(\frac{R_{1,2}}{R_\odot}\right) - 10 \log \left(\frac{T_{1,2}}{T_\odot}\right)
\]

and

\[
L_{1,2} = \left(\frac{R_{1,2}}{R_\odot}\right)^2 \left(\frac{T_{1,2}}{T_\odot}\right)^4
\]

Assuming that \(T_{\text{eff1}} = 6705\, \text{K}, T_{\text{eff2}} = 6544\, \text{K}\) and \(T_\odot = 5778\, \text{K}\), the bolometric magnitudes are \(M_{\text{bol1}} = 3.278 \pm 0.024\) and \(M_{\text{bol2}} = 4.754 \pm 0.024\), while the solar luminosities for the primary and secondary are \(L_1 = 3.88 \pm 0.09 L_\odot\) and \(L_2 = 1.00 \pm 0.04 L_\odot\), respectively.

The distance to TYC 0163-0488-1 \((438 \pm 5\) pc) was estimated using the distance modulus Equation (10) corrected for interstellar extinction. In this case \(V\) mag at Min II \((m = 11.57 \pm 0.01)\) is defined when the primary totally eclipses the secondary, and \(M_V\) is the absolute magnitude derived using the bolometrically corrected magnitude \((M_{\text{bol1}} - BC)\). The interstellar extinction \((A_V = 0.05425 \pm 0.00155)\) was determined from \(E(B-V)\), the color excess previously described in section 3.3 where \(E = 3.1\).

\[
d(\text{pc}) = 10^{0.2(m-M_V-A_V)/5} / 5
\]

Another value for distance \((399 \pm 36\) pc) was calculated according to the empirical expression (11)

\[
\log D = 0.2 \times V_{\text{max}} - 0.18 \times \log P - 1.6 (J-H) + 0.56
\]

derived by Gettel et al. (2006) from a ROTSE-I catalog of overcontact binary stars where \(D\) is distance in parsecs, \(P\) is the orbital period in days, \(V_{\text{max}} = 11.17 \pm 0.01\), and \((J-H)\) is the 2MASS color for TYC 01963-0488-1. The combined mean distance to this system is therefore estimated to be \(419 \pm 18\) pc.

4. Conclusions

CCD-based photometric data captured in B, V, and Iₜₚ passbands produced five new times-of-minimum for the W UMa binary system TYC 01963-0488-1. A first epoch \((HJD_0)\) linear ephemeris for TYC 01963-0488-1 was established for this system; many more years of data will likely be required to determine whether there are any secular changes in orbital period. The weight of evidence from this study and other surveys suggested that the effective temperature of the most luminous star is 6705 K and corresponds to F2V-F3V spectral class. The greatest challenge to definitive Roche modeling of the rapidly expanding catalog of newly discovered W UMa binaries is the absence of published RV data to unequivocally determine a mass ratio \((q)\) and subtype \((A\) or \(W)\). Fortuitously this system experiences clearly defined total eclipses which help constrain a photometrically determined mass ratio result \((q \sim 0.25)\). Since maximum light at \(\varphi = 0.25\) and 0.75 was not equal, a cool-spot solution was necessary to achieve the best Roche model fits for TYC 01963-0488-1. Similar LC behavior observed between 1999 and 2000 suggests that this system has an active photosphere. Until which time RV data become publically available, these Roche model fits and any absolute parameters derived for this W UMa binary are subject to some uncertainty. The sum total of all data collected thus far suggests that TYC 01963-0488-1 is most likely an A-type W UMa variable located over 400 pc from our home planet. Public access to any data associated with this research can be obtained by request (mail@underoakobservatory.com).

5. Acknowledgements

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