Astronomy C: 2012 National Exam Solutions

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Note: These solutions are for the computational (physics) questions only. All values given are somewhat approximate, the median of a range of answers allowed. I stop listing conversion factors after they have been used once for brevity.

Section B:
Question 1

b) Here use the following:

$$a = r\theta \tag{1}$$

Where a is the separation between the stars, r is the distance to the system (8.6 ly), and θ is expressed in radians (3600" in 1°, $\pi/180$ radians in a degree). Then we may use the following:

$$\frac{a^3}{T^2} = M + m \tag{2}$$

Where T is the period of the orbit in years, we have re-expressed a in AU (63,240 AU/ly) and M and m are the masses of the two stars in solar masses. Solving this, we get T = 75 years.

c) Here we assume the orbits are circular, and hence the distances from the common center of mass (r_A, r_B) can be expressed as a ratio:

$$\frac{r_A}{r_B} = \frac{M_B}{M_A} \tag{3}$$

Plugging in the masses, we can derive the ratio of the two distances to the center of mass, which when setting $r_A + r_B$ = 8.6ly we find r_A = 8 AU.

d) Here we use the Stefan-Boltzmann relationship:

$$\frac{L_A}{L_B} = \frac{R_A^2}{R_B^2}\frac{T_A^4}{T_B^4} \tag{4}$$

Plugging in and solving for the temperature of Sirius B, we get T = 32,000 K.

Question 4

c) Again we use the Stefan-Boltzmann relationship, adjusted as follows:

$$\frac{L_{HD62166}}{L_\odot} = \frac{R_{HD62166}^2}{R_\odot^2}\frac{T_{HD62166}^4}{T_\odot^4} \tag{5}$$

Plugging in values, and knowing that the Sun has a temperature of $\approx$ 5770K, we get $T_{HD62166}$ = 190,000 K.

Question 6

d) In part (b) the orbital period of the binary was found to be $\approx$ 300 seconds. Using this, and Equation (2) through unit conversions of 9.30 x 10^7 miles/AU and 3.156 x 10^7 s/yr, we get M_{Tot} = 1.72 $M_\odot$.

Question 13

f) Here we use the standard expression for kinetic energy:

$$E_k = \frac{1}{2}mv^2 \tag{6}$$

Plugging in values, and making proper conversions 1 erg = 10^{-7} Joules and 2 x 10^{30}kg = 1 solar mass, we get $M_{remnant}$ = 2.22 $M_\odot$.

g) Here we use Equation (1) and the conversion 1° = 60' to find an angular radius of Tycho's SNR = 3.2 pc. Using the expansion velocity from part (f), and making the conversion 1 pc = 3.086 x 10^{13} km, we find t_{Tycho} = 348 years.

Section C:
Question 1

a) This object is T Tauri. Reading off from the table, and using the conversion 3.26 ly = 1 pc, we get D_{TTauri} = 587 ly.

b) This is Mira. Here we use the distance modulus:

$$m - M = 5log\frac{d}{10pc} \tag{7}$$

Where m is apparent magnitude, M is absolute magnitude, and d is distance in parsecs (130 pc for Mira). We convert luminosity in solar units to M as follows:

$$M = 4.83 - 2.5 log \frac{L}{L_{\odot}} \tag{8}$$

Using all of this, we find $m_{Mira} = 10.6$.
c) This is RR Lyrae. We use Equation (8) to find $M_{RRLyr} = 0.70$.
d) This is M15. We use Equation (1) and the given distance to find $R_{M15} = 8.2$'.
e) This is NGC 2440. We convert 1230 parsecs to a distance of 3.8 x 10^{16} km.

Question 2

This is a Type Ia SNe with a peak apparent magnitude of ≈ 15.1. All Type Ia SNe have a peak absolute magnitude of ≈ -19.5, and using the distance modulus (Equation 7) we calculate a distance of 83 Mpc.

Question 3

This is an RR Lyrae variable with an average apparent magnitude of ≈ 15.5. All RR Lyrae variables have an average absolute magnitude of ≈ +0.75. Using Equation 7, we calculate a distance of 8.9 kpc. The conversion for parallax is:

$$d(pc) = \frac{1}{p('')} \tag{9}$$

Where p is the parallactic angle in arcseconds. Hence we find a parallax of 1.12 x 10^{-4}".

Question 4

We use the age and expansion velocity of the nebula to calculate its radius to be 5.11 milliPc. Using Equation 1, we find a distance of 2.0 pc.

Question 5

a) We use Equation 9 to find the distance to Object D of 100 pc. Next we use Equation 1 to find the diameter of Object D to be 50 pc.
b) Here we use the following relationship:

$$\frac{r_E}{r_F} = \frac{v_E}{v_F} \tag{10}$$

Looking at the numbers, we see $v_E/v_F = 2$.
c) Here we equate the force due to circular motion $F = mv^2/r = GMm/r^2$ to get the following relationship for v:

$$v = \sqrt{\frac{GM}{r}} \tag{11}$$

Where G is the universal constant of gravitation, M is the mass of the black hole (10,000 $M_{\odot}$), and r is the distance from Star G to the black hole, 10 light years. Plugging this in, we get $v_G = 3.8$ km/s.

Question 6

a) Reading off the light curve we find its period to be 20 years. Knowing that the two stars are separated by 20 AU, we use Equation 2 to find $M_1 + M_2 = 20\ M_{\odot}$.
b) Here we must find the orbital velocity of star H using the redshift and blueshift given, and then use the ratio of orbital velocity to mass ($V_H/V_J = M_J/M_H$) and the result from part (a) to find the mass of Star H. To find the velocity of Star H we use:

$$\frac{\lambda_H - \lambda_{lab}}{\lambda_{lab}} = z = \frac{v_H}{c} \tag{12}$$

Where λ_H may be either (the maximum red or blue-shifted) H-alpha line wavelength, λ_{lab} is 656.3nm, and c is the speed of light. Plugging in values, we find $V_H = 91.4$ km/s. Applying this to the ratio of velocity to mass and knowing that the orbital velocity of Star J is 5 km/s, we find $M_H = 1M_{\odot}$.
c) This question was likely the trickiest on the exam. We already found the mass of Star H in part (b), and hence to find the density we must compute its radius. The way to do this is to use the velocity of Star J and half the time it takes to eclipse Star H-reading off from the light curve, this half-time is 1.1 x 10^3 seconds. Now we use r=vt to compute the radius of Star H, 5500 km. We then calculate the density as follows:

$$\rho = \frac{M}{\frac{4}{3}\pi r_h^3} \tag{13}$$

And plugging in values we get $\rho_H = 2.9$ x 10^9 kg/m^3.
d) It is evident from the ≈ 1 solar mass, small radius and hence high density of Star H that it is a white

dwarf.

Question 7

Here we use spectroscopic parallax for main-sequence stars. We find the spectra to be that of a B5 star, and reading off from a table of data for main-sequence stars of a range of spectral types (a great one may be found in the appendices of Carroll and Ostlie's "Introduction to Modern Astrophysics") or an H-R diagram with B-V as an x-axis, we find an absolute magnitude in the range of -2 to -2.5 and hence a distance of $\approx$ 4500 parsecs using Equation 7.

Question 8

Here we again use spectroscopic parallax, and a table or H-R diagram that you hopefully had (if not, you should put it in for next year!) in your multitude of reference material. This star is a M0 star, and reading off from your reference you should get a B-V in the range of 1-1.8.

Question 9

Angular velocity is related to orbital velocity by the following:

$$\omega = \frac{v}{r} \tag{14}$$

Where r is the distance to the center of mass of the system, 0.5 AU. We use the conversion 1 AU = 1.5 x 10^{11} m, and the relationship for circular orbital velocity:

$$v = \frac{2\pi r}{T} \tag{15}$$

Where T is the period of orbit, 20 days. Applying these formulae and conversion factors, we find an angular speed of Star M of $\omega_M = 3.6$ x 10^{-6} radians/second.

Question 10

a) Here we use Wien's Law:

$$T = \frac{0.0029mK}{\lambda} \tag{16}$$

Where T is temperature in Kelvin and λ is the peak wavelength of radiation emitted in meters. Plugging in values, we get T_N = 14,500 K.

b) Here we use the Stefan-Boltzmann relationship (Equation 5) and plugging in values we get $L_N = 1000\ L_\odot$.

If you have any questions about the exam, please send me an email, I'd be happy to help! Good luck with the 2012-2013 school year and Science Olympiad season!