PERIOD ANALYSIS OF W DRACONIS

John Greaves
15 Borrowdale Walk
Northampton, NN3 6PW
United Kingdom

Received March 1999; revised January 2000

Abstract

The period variations of the Mira variable W Draconis are analyzed and compared with those of several other stars.

1. Introduction

W Draconis is an M3e–M4e III Mira star noted by Wood and Zarro (1981) as being of constantly increasing period. Mira stars can appear to systematically change in period, whereas in fact they are exhibiting random drift, so Lombard and Koen (1993) further investigated this star's period variation using statistical techniques upon the AAVSO archival data set (Mattei 1998), confirming the reality of the changes.

2. Period Analysis

Figure 1 shows an O-C plot for data ranging from ~ JD 2423000 to 2451000. This plot was generated using observed maxima derived using data from the AFOEV (1998) and VSOLJ (VSNET 1998) ftp sites, with some maxima taken from Mattei *et al.* (1990) to cover gaps in the data between 1950 and 1960. A parabolic deviation in an O-C plot indicates a constant rate of period variation, and as can be seen from the plot the data are well fitted by a quadratic expression. The product moment correlation coefficient (R^2) is 0.983, which is strong evidence that W Dra has been regularly varying in period throughout its observational history.

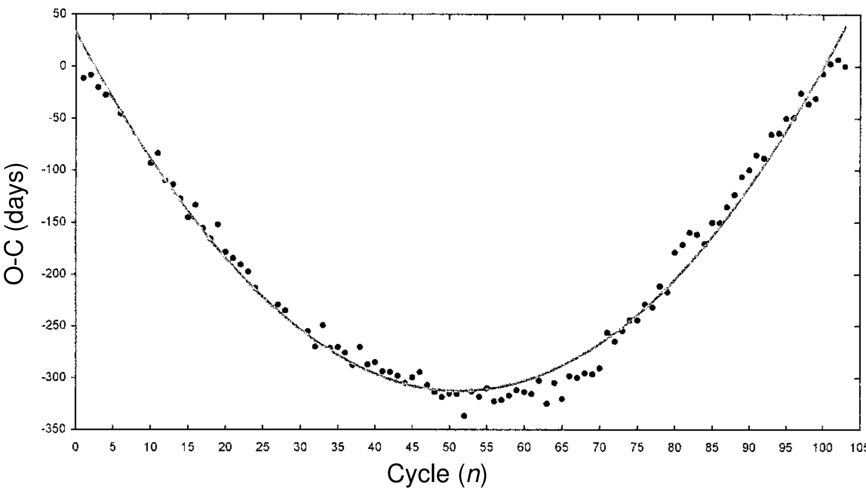

Figure 1. O-C plot for W Draconis from ~JD 2423000 to JD 2451000. The superposed line is a quadratic best fit of an R^2 relation of 0.983 when compared to the data.

The O-C plot was generated using an average test period derived from $(t_n-t_0)/n$ where t_n is the Julian Date of the last maximum at cycle n and t_0 that of the initial maximum. This enables any variation from this mean within an O-C plot to be tested for statistical significance. The methods used here are the chi-squared method of Sterne and Campbell (1936), the Span Length Test of Isles and Saw (1987), and the correlation of the cycle lengths, offset by two positions, as outlined by Lloyd (1991). The results are: span test statistic equals 24.79 for 103 cycles (though with some gaps); published tables give a threshold of 16.2 for $p = 0.005$, lag2 cycle length correlation $p = 0.000015$; and Sterne's chi-squared test not quite as good as these two latter at $p = 0.0295$; all for the null hypotheses of random variation within the data. In other words, these three independent tests give strong indications that the period of W Dra is indeed variable.

However, Lombard and Koen (1993) also note that a good indication of the behavior of a Mira's period over time can be found via a plot of the intermaximal cycle lengths (usually loosely referred to as the "periods") superposed by a moving average line. Figure 2 shows such a plot based on the "periods" derived via the above O-C and with a seven-point moving average line superposed. This line appears to confirm the canonical view over most of its length for an increasing period over time, but becomes somewhat flatter towards the end of the plot.

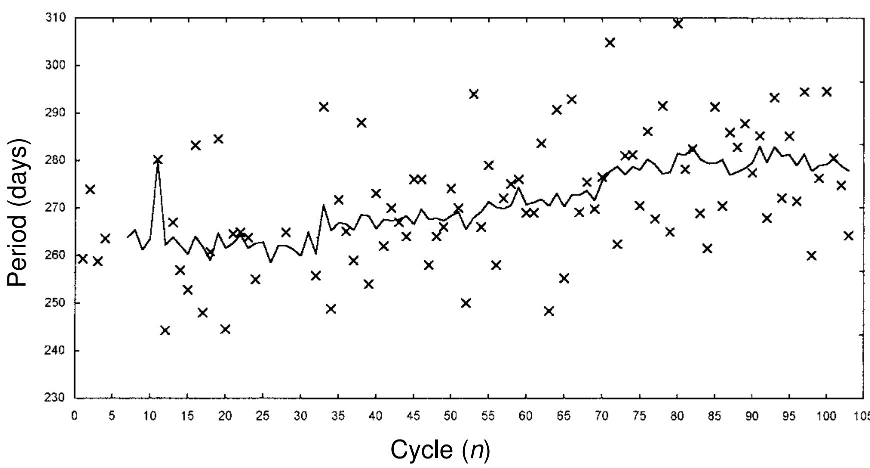

Figure 2. Plot of intermaximal cycle lengths ("periods") against cycle number for W Dra from ~ JD 2423000 to JD 2451000. The superposed line is a seven-point moving average which can be seen to first follow an increasing trend in terms of period before levelling off somewhat.

3. AMPSCAN Analysis

Further investigation was desired, and we remedied the sparsity of data during certain periods by the addition of AAVSO data for this star (Mattei 1998). This data set allowed the author to investigate matters further via the AMPSCAN procedure. Howarth (1991) introduced this methodology in an investigation of the complex behavior of the semiregular variable W Cygni. Basically, a form of Fourier decomposition over time is utilized to measure the phase stability of a light curve over its history. An input period is used and a sinusoid based on this period is fitted against the light curve via a moving window.

If the true light curve has a constant period, then a plot of phase against time will be linear. The input period does not even need to be accurate, as a slightly incorrect value will still lead to a linear plot whilst sloping at an angle. A parabolic plot of phase over time indicates a constant rate of change of period, directly analogous to the case for an O-C plot.

One great advantage of the AMPSCAN methodology is that it samples the entire light curve, and not just some arbitrarily appointed representative point, which in many ways is all that the time of maximum (and/or minimum) represents. It is also consequently free from the measurement errors that affect the derivation of times of maxima and/or minima. As the method basically "scans" the amplitude of the light curve over time it can be prone to breaking down either when a variable is at a state of very low amplitude (as can happen in semiregular stars from time to time) or when data are very sparse. Times when the latter is the case are usually well evident from the light curve. An added bonus of the method is that it also generates a plot of semi-amplitude over time: amplitude over time is something that is not often addressed in variable star analysis.

4. Results

With the improved data set provided by the AAVSO archive it was possible to perform AMPSCAN analysis on this star. First the data were averaged into 5-day means. The next step was to provide a suitable input period for the AMPSCAN procedure, and this is normally done via a periodogram. In this case a *semi-amplitude* spectrum was generated (as opposed to the usual *power* spectrum) via an algorithmic formulation of Howarth's (1991), and is shown in Figure 3.

It can be seen that the main period around 0.0036 cycle per day (cpd) is spread over several peaks. A pair of peaks around the 0.0063 cpd point are the beat period between

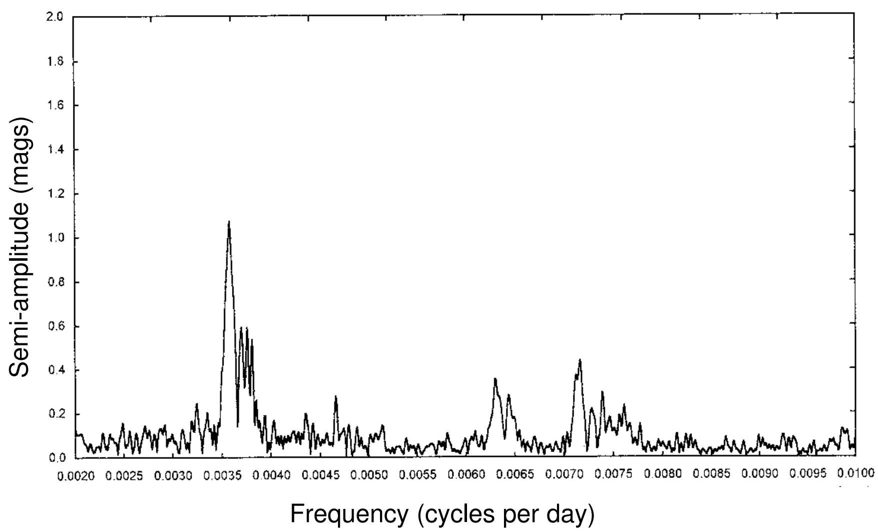

Figure 3. Semi-amplitude spectrum for the full W Dra data from ~JD 2419000 to JD 2451000. The main peak of semi-amplitude 1.1 magnitudes denotes a period of 279.2 days. The peaks surrounding 0.0072 cpd are first harmonic values due to the fact that W Dra's light curve is not exactly represented by a perfect sinusoid. Those surrounding 0.0063 cpd represent the beat alias between the main peaks and an annual observing regime.

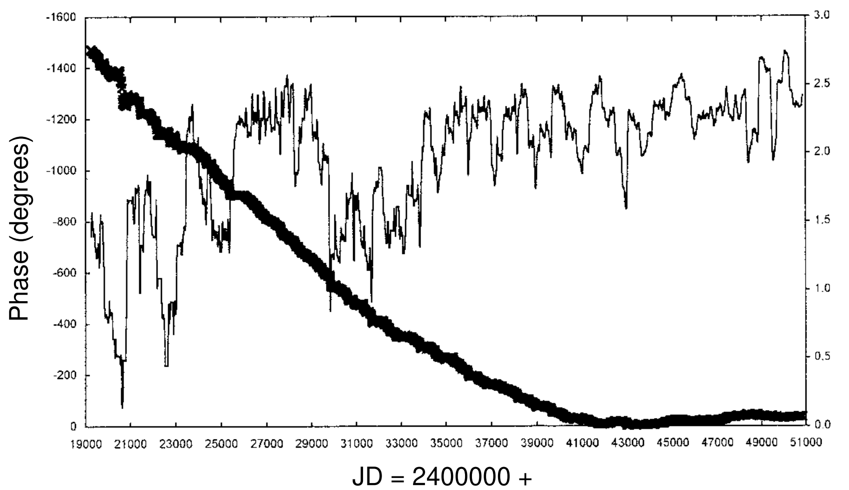

Figure 4. Phase in degrees (thick line) against Julian Date (left *y* axis) and semi-amplitude (thin line) against Julian Date (right *y* axis) for W Dra. The AMPSCAN input period was 279.2 days, and it can be seen that prior to JD 2442000 phase is not stable about this period. After that date phase is fairly constant about the zero value.

the main period and the annual alias which exists due to the time regime of the observations. Those near 0.0072 cpd are at the first harmonic and reflect the fact that W Dra's mean light curve is not actually representable by a single pure sinusoid. The main peak is at about 1.1 magnitude semi-amplitude and represents a period of 279.2 days. This value was then input into the AMPSCAN procedure and the 5-day means were analyzed at a step size of 10 days, the resultant plot being shown in Figure 4.

AMPSCAN itself outputs phase as a relative value in comparison to the phase at the midpoint of the data run, and as a consequence of the mathematics folds phase at 0 and 360 degrees. Here the phase is adjusted such that the linear portion of the plot is at around 0 degrees and increments of 360 degrees are subtracted on wrap around for illustrative purposes. It should be noted that we are looking at the *relative* behavior of phase over time, and the absolute value at any point is not strictly relevant. The plot of semi-amplitude over time is also included.

The thing to note from this plot is that since around JD 2442000 phase has been fairly flat, i.e., there has been little deviation from the input period of 279.2 days. Prior to that time, the phase shows a shallow deviation from this value. This reflects the fact that the period in the past was not exactly 279.2 days. A decrease in phase value denotes a decrease in period, and an increase in phase value an increase in period: the phase axis has been labeled inversely according to a convention that allows AMPSCAN phase-against-time plots to be morphologically analogous to O-C plots (see Greaves and Howarth 2000 for examples of the behavior of other Mira stars).

This plot provides an objective means of splitting a data set. With O-C plots it is difficult to say when a deviation is indicative of a true variation and when it is merely a consequence of the procedure or fluctuations of the times of maxima. From an AMPSCAN we derive a period which has its phase stability tested. In the case of W Dra we found that the period was only reasonably constant in the latter part of the observations. Prior to then, other periods held sway.

This now allows the splitting of the data into two subsets. The first subset consists of the data prior to JD 2442000 (roughly 1973), and its semi-amplitude spectrum is plotted in Figure 5. Here we have a broad peak of relatively low semi-amplitude, indicating the existence of several periods. Such periodograms "lump" all periods within the data together, irrespective of where they occurred. Figure 6 shows the periodogram for data since JD 2442000: here we have a fairly tight peak of nearly 2.0 magnitudes semi-amplitude accompanied only by its first harmonic and the annual beat alias. This can be interpreted as representing a stable period throughout this time; the derived full amplitude of nearly 4 magnitudes represents the mean variation of this star quite well (remembering that catalogue values quote isolated extreme values for the range).

5. Discussion

Wood and Zarro (1981) provide the standard Helium Shell Flash model for long-term period variation in Mira variables. Although the time scales derived via this model seem appropriate for stars like R Aquilae, which has been declining in period constantly since its discovery in 1856 (*e.g.*, Greaves 1998; Greaves and Howarth 2000), there are difficulties when it comes to the other stars usually quoted. It is often opined that the list of four stars known to be of regularly-varying period (out of the total of ~ 300 well known Miras) is just the right sort of amount to fit aspects of this model: two stars of constantly-declining period (R Aql and R Hydrae); one of constantly-increasing period (W Dra); and one that has just begun to decline in period (T Ursae Minoris: Gal and Szatmary 1996; Foster 1996; Greaves, unpublished).

Of these four, we only have one star that can be shown to have been constantly changing in period throughout its observational history, with Greaves (1998) giving a quadratic fit to nearly a century of BAA,VSS data for R Aql that can be extrapolated back to a period of approximately 340 days at its 1856 discovery date, which is in the right region (348 days is given in the 5th Edition of the *Bright Star Catalogue* (Hoffleit *et al.* 1996)). W Dra *stopped* changing in period in 1973. T UMi *began* to change in period around 1982, and has declined from a period of 312.5 days to around 280 days since that time (Greaves, unpublished), whereas it took R Aql 90 years to decline from a similar 315-day period to its current one of around 280 days.

These "coincident" stopping and starting events do not fit well with the hundreds-of-years time scales for Wood and Zarro's models, *i.e.*, it is strange that out of the four events known we should witness the start and end of two of them within the space of ten years. The situation is compounded by the case of R Hya, which ceased to vary systematically in period around 1940 (Greaves 1998; unpublished). Unlike W Dra, the case for R Hya is so evident that it can easily be shown merely by splitting the traditionally-plotted O-C for this star (*e.g.,* Wood and Zarro 1981) at the "bend," and using statistical tests on the two resultant subsets, without recourse to new procedures like AMPSCAN. Further, if periodograms are generated for these two subsets independently, it can be seen that the data since 1940 are well represented by a fairly constant period of 389 days (as is indeed the value given in the 4th Edition of the *General Catalogue of Variable Stars* (Kholopov *et al.* 1985) (GCVS) and very close to the value of 388 days given in the 3rd Edition of the GCVS (Kukarkin *et al.* 1969)).

Wood and Zarro (1981) themselves note that prior to 1708 R Hya's period was not in decline, with the onset lying somewhere between this 1708 and 1730. Given that their models predict a 950-year interval of period decline for this star, it is interesting to note that the evidence rather suggests that this interval only actually lasted between 210 and 230 years. It is also interesting that the literature is often ready to note that R Hya declined from a period of around 500 days at the turn of the 18th Century to around 400 days at the turn of the 20th, *i.e.*, roughly a hundred days in two centuries, but seems to have missed the fact that at the turn of the 21st Century the star has a mean period of about 390 days, *i.e.,* a decline of barely ten days in one century.

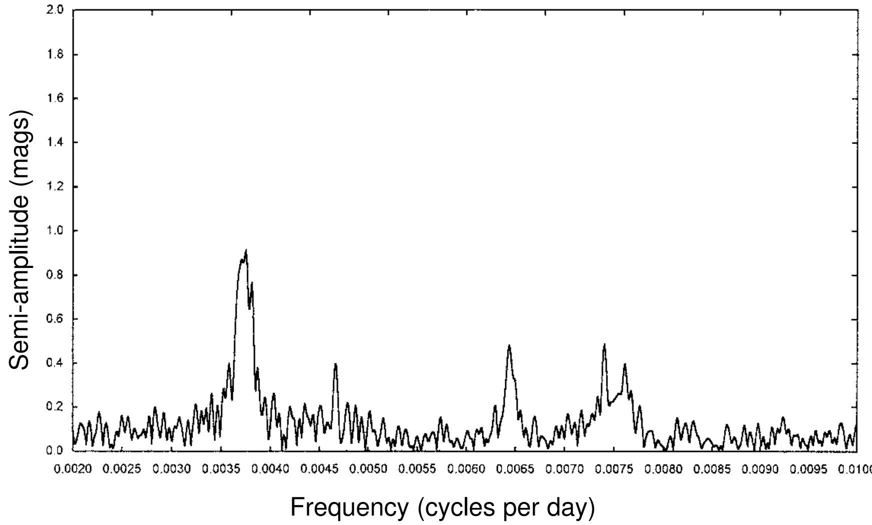

Figure 5. Semi-amplitude spectrum for the W Dra subset of data *up to* JD 2442000. A broad peak of low semi-amplitude shows that the periodicity throughout this interval is spread over many values. Other peaks represent beat aliases and harmonics.

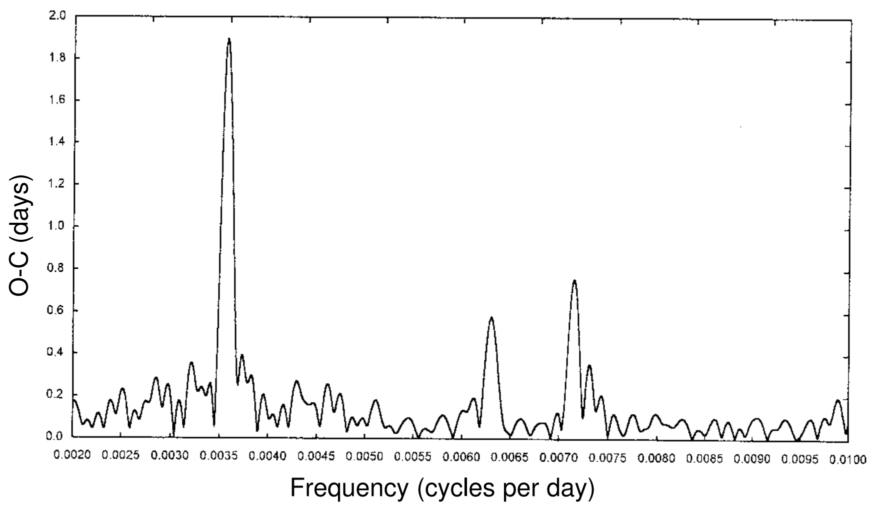

Figure 6. Semi-amplitude spectrum for the W Dra subset of data *since* JD 2442000. A tight peak of relatively large semi-amplitude at around 279 days period shows that periodicity throughout this interval is well represented by this period. Other peaks represent beat aliases and harmonics.

6. Summary

W Dra was shown to have ceased increasing in period since about 1973, and to have remained at a fairly stable mean period of 279 days since then. The AMPSCAN procedure was shown to be an effective means of objectively appraising the behavior of a Mira star's light curve over time by adding an extra dimension to periodogram analysis. Difficulties with the standard model for such stars were outlined.

7. Future Work

The raw material for any investigation into the long term period behavior of W Dra is crucially dependent upon the work of visual observers. All observers of this star are gratefully thanked for their past efforts and are encouraged to continue to observe it. It is important to know whether this star really has settled down to a regime of fairly constant mean period, or whether it will start to change in period again, and if so, in which direction!

8. Acknowledgements

The author is grateful to John Howarth for advice and discussion on aspects of his "moving window fourier decomposition technique" (nowadays usually called AMPSCAN for convenience); Chris Lloyd for providing a DOS executable that calculates Sterne's statistics; Janet Mattei for providing just the right amount of data for placing a preliminary analysis of this star on a much firmer footing!

References

AFOEV 1998, ftp site: ftp://cdsarc.u-strasbg.fr/pub/afoev/
Foster, G. 1995, *Astron. J.*, **109**, 1889.
Gal, J., and Szatmary, K. l995, *Astron. Astrophys.*, **2977**, 461.
Greaves, J. 1998, *J. British Astron. Assoc.*, **108** (6), 320.
Greaves, J., and Howarth, J. J. 2000, *J. British Astron. Assoc.*, **110** (3), 131.
Hoffleit, D., *et al.* 1996, *The Bright Star Catalogue,* 5th Preliminary Ed., NASA ADC CDROM Vol 1.
Howarth, J. J. 1991, *J. British Astron. Assoc.*, **101** (2), 101.
Isles, J. E., and Saw, D. R. B. 1987, *J. British Astron. Assoc.*, **97** (2), 106.
Kukarkin, B. V., *et al.* 1969, *General Catalogue of Variable Stars,* 3rd Ed., Moscow.
Kholopov, P. N., *et al.* 1985–1988, *General Catalogue of Variable Stars*, 4th Ed., Moscow.
Lloyd, C. 1991, *J. British Astron. Assoc.*, **101** (1), 46.
Lombard, F., and Koen, C. 1993, *Mon. Not. Royal Astron. Soc.*, **263**, 309.
Mattei, J. A. 1998, visual observations from the AAVSO International Database.
Mattei, J. A., Mayall, M. W., and Waagen, E. O. 1990, *Maxima and Minima of Long Period Variables 1949–1975*, AAVSO.
Sterne, T. E., and Campbell, L. 1936, *Ann. Harvard Coll. Obs.*, **105**, 459.
VSNET 1998, ftp site: ftp://ftp.kusastro.kyoto-u.ac.jp/pub/vsnet/VSOLJ/database/stars/
Wood, P. R., and Zarro, D. M. 1981, *Astrophys. J.*, **247**, 247.