Discrete Fourier Analysis of the Light Curve of S Persei

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Abstract A discrete Fourier analysis was performed on the validated S Persei visual light curve data as obtained from the American Association of Variable Star Observers (AAVSO). These observations span just over a century, from February of 1903 to July of 2003. This analysis was an attempt to find the fundamental periods of the variability of the red supergiant S Persei. Inspection of the S Per light curve indicates a likely complex combination of sinusoids of differing periods. Using Fourier analysis, four periods of various relative strengths were extracted from these data: 745, 797, 952, and 2857 days. Although some of these periods are similar to earlier results, they seem to indicate a more complex result than has previously been determined.

1. Introduction

S Persei (designation 0215+58) is a cool, red supergiant with semiregular variability, and is listed in the *General Catalogue of Variable Stars* (GCVS; Kholopov *et al.* 1985) as an SRc variable of spectral and luminosity type M3Iae–M7. It is found at a distance of 2.55 ± 0.26 kpc (Yoshiharu 2003) and is about as massive as twenty suns. It was first discovered by A. Krueger in 1872 (Krueger 1874), and has been regularly observed since then. Starting in February of 1903, the AAVSO has catalogued 24,260 observations of S Per. These observations correspond to Julian Dates 2416160 (Feb. 14, 1903) through 2452848 (Jul. 27, 2003) Figure 1 shows the visual light curve data for S Per, with all CCD filter or other non-visual observations removed. The first 35 of these observations were also removed for this analysis, as they predate the use of standardized comparison stars.

H. H. Turner (Peek and Turner 1904) made the first attempt to interpret the odd fluctuations of S Per, but with only a few years of regular observations. Turner hypothesized the light curve was made up of three periods of 840, 1120, and 3360 days. 35 years later, T. E. Sterne performed a new analysis, finding only two periods of 810 and 916 days (Campbell 1939). Periods of 807 and 968 days were derived for S Per from AAVSO data along with analyses of several other variable red supergiants by R. Stothers and K. Leung (1971). J. Isles found periods of 829 and 947 days (Isles 1973), and later still, H. Smith found only two periods as well, this time 825 and 940 days (Smith 1974), using a power spectrum analysis. Most recently, I. Howarth's periodogram analysis indicated one main period of 822 days (Howarth 1976), though other peaks were present in his analysis. However, using only two periods will produce a more dominant beat frequency than is apparent in Figure 1. This paper takes advantage of the additional thirty years of regular observation since then, using data validated by the AAVSO organization.

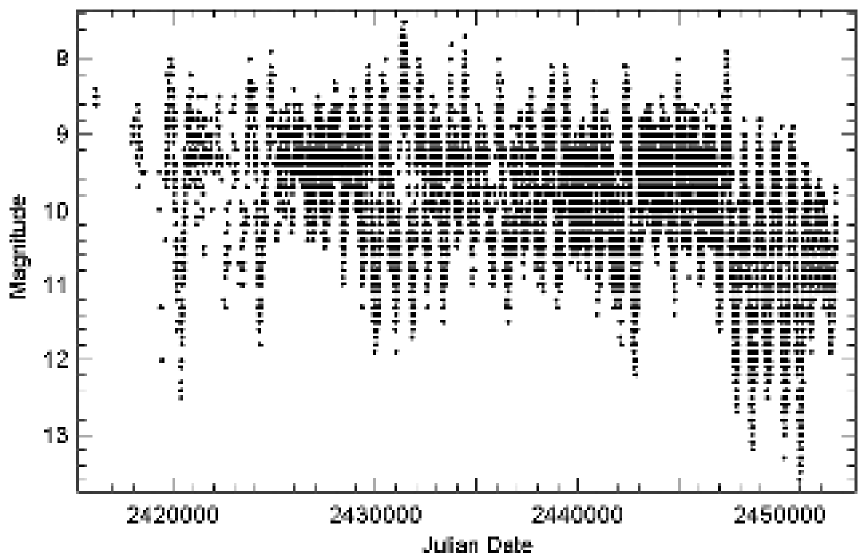

Figure 1. Light curve for S Persei.

Table 1. Periods and amplitudes for S Persei as determined by various authors.

Author (Reference)				
Peek and Turner (1904)	—	840d	1120d	3360d
Sterne (Campbell 1939)	—	810d	960d	—
Stothers and Leung (1971)	—	807d	968d	—
Isles (1973)	—	829d	947d	—
Smith (1974)	—	825d	940d	—
Howarth (1976)	—	822d	—	—
This Paper	745 ± 24d	797 ± 24d	952 ± 24d	2857 ± 81d
Magnitudes (this paper)	0.9	0.56	0.54	0.4

2. Data interpolation

We used a Fourier algorithm that requires even sampling. Since these data are, for the most part, unevenly spaced, we interpolate to create a sampling of data that is almost identical to the original, but that can be subjected to Fourier analysis. Discussions of treating unevenly sampled data are presented by Schwarzenberg-Czerny (1996), among others.

In the MATHEMATICA analysis package, the function "Interpolation[ ]" performs the necessary task. This command creates an "Interpolating Function," which can then be used to create a new data set. The option "InterpolationOrder" specifies the order of the interpolating function. For this analysis, first order (linear) interpolation is used, made possible by the copious amounts of data that are closely spaced in time. This Interpolating Function acts like any other analytical function in MATHEMATICA for the range over which it is defined, and interpolates a value if there is no value specified by the original data set. By writing a simple "for loop" (just as in C, C++, or Fortran, etc.), we can take advantage of the Interpolating Function to create a subsampled data set evenly spaced in time. A time step of five days was chosen.

To account for the effects of interpolation, the analysis was also run using a third order interpolation to check the first order results. We found it did not affect the outcome in any way. Different time steps were also tested. A time step of one day revealed the same four periods as the five-day step, differing only by tens of minutes, and the same was true for a ten-day step. A time step of thirty days also showed all four periods, differing by only, at most, about 5 days from those found in section 3. Lastly, a 100-day step was tried, once again revealing all four periods, only this time differing from those in section 3 by up to about 50 days. In each case, the relative magnitudes of the peaks and the depths of the valleys between those peaks were not affected. Hence, interpolation of the data does affect the Fourier analysis, but not by very much. In fact, the differences caused by different time steps are almost all well within the calculated error from the Fourier spectrum, as discussed in the next section.

3. Fourier analysis

Discrete Fourier analysis is a common method of finding the inherent frequencies in cyclic physical systems, including electrical circuit analysis, mechanical vibrations, image compression, and orbital dynamics (see, for example, Hunter 2002 or Ransom *et al.* 2002). The Fourier transform of the light curve data is shown in Figure 2, as produced by MATHEMATICA. The Fourier spectrum was independently verified using a Fast Fourier Transform (FFT) in the Interactive Data Language (IDL) software. The large peak at zero frequency must be disregarded, as it is an artifact of the Fourier transform created by the non-zero average magnitude of the light curve data. Using similar logic to Smith (1974), peaks over sixteen times the average of the Fourier

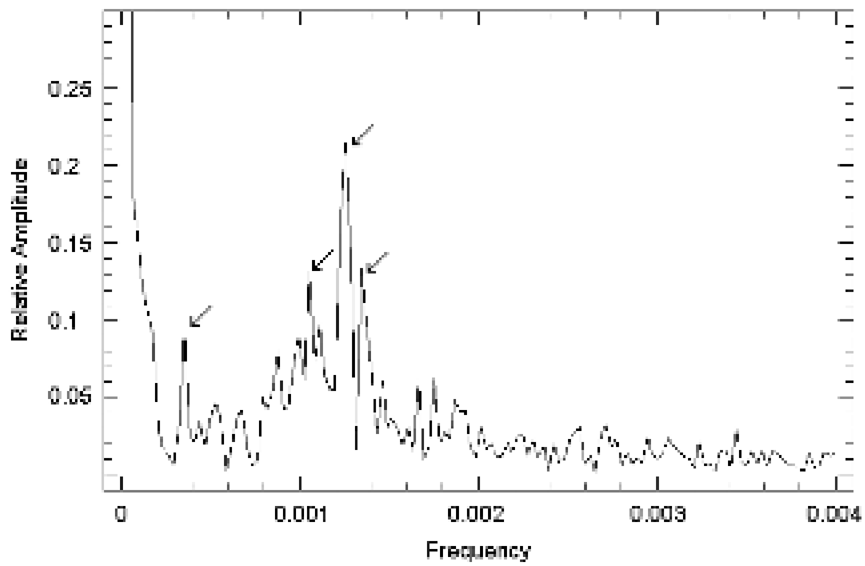

Figure 2. Normalized spectrum of S Per light curve. Marked peaks are those which pass statistical analysis.

power, equivalently, fourtimes the average of the amplitude, can be considered to be real periods with some confidence, instead of just statistical fluctuations. Still other peaks are eliminated by noting that they do not exceed one standard deviation of the Fourier data relative to those minima surrounding them. After this statistical analysis of the Fourier transform data, four peaks, and thus four periods, stood out in the light curve. The four periods are, from strongest to weakest: 797 ± 24d, 745 ± 24d, 952 ± 24d, and 2857 ± 24d. Interestingly, we should note that the 2857- and 952-day periods are harmonics of each other, that is, even multiples to three decimal places. These periods have respective approximate amplitudes of 0.9, 0.56, 0.55, and 0.4 magnitude. Using a non-linear best fit function in Mathematica to find the relative phase shifts of the four periods, a simulated light curve was created to test against the real data.

Because earlier analyses have picked out only two periods, the two strongest period components were used to create another simulated light curve, to contrast with the four-period curve. The plots of these two simulated curves offset against the real curve are shown in Figures 3 and 4.

Using Mathematica to find the averages of the errors between the two simulated curves and the actual light curve, we find that the four-period curve is a much better approximation and follows the shape of the empirical data rather closely. While the two-period curve addresses the overall shape of the light curve, it cannot account for as many fluctuations in the empirical data as four periods. Inspection of Figures 3 and 4 indicates that the two-period curve has a much more apparent beat frequency

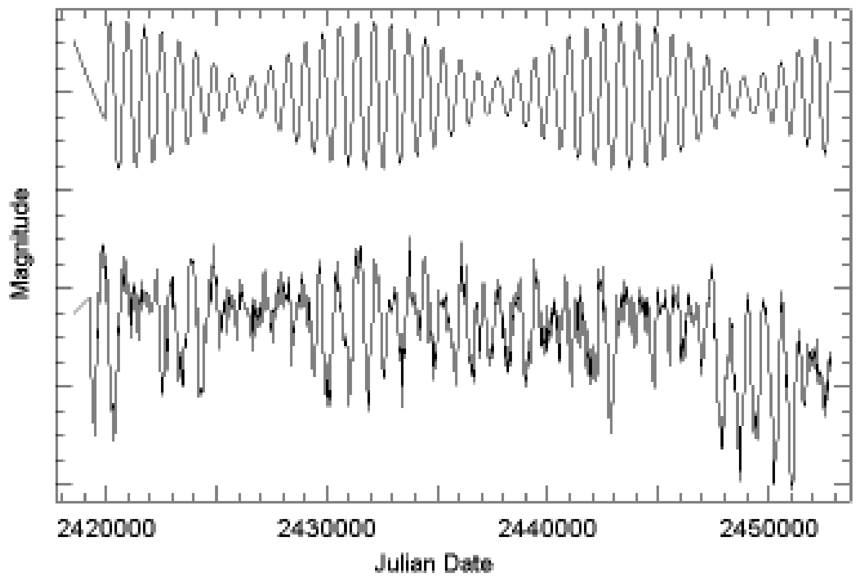

Figure 3. Simulated two-period curve (top) and actual curve.

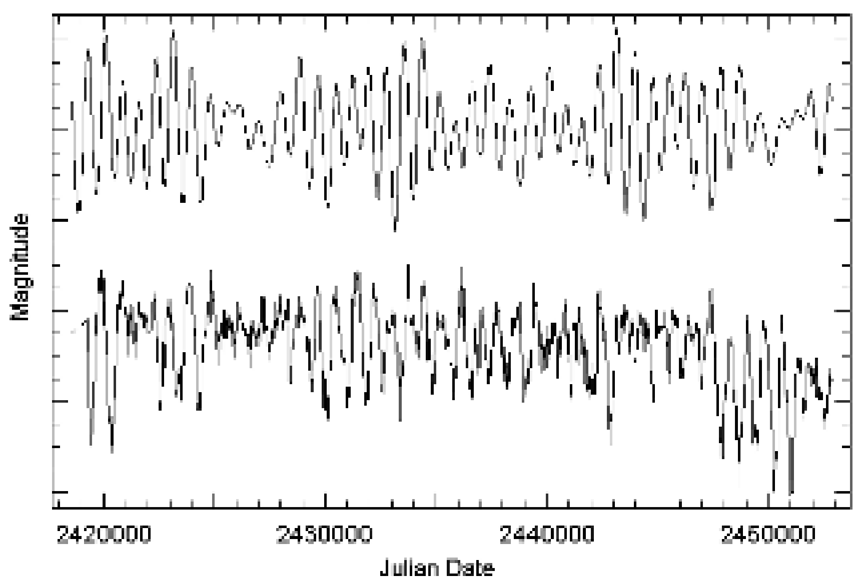

Figure 4. Simulated four-period curve (top) and actual curve.

than is found in the empirical data, and that the four-period curve accounts for more of the real light curve's irregularity.

4. Recent years

In the past couple of decades, the light curve of S Per has shown a fading in average magnitude. This is easily seen in Figures 1 and 5. Using only the portion of data after Julian Date 2447000, another Fourier analysis revealed a major peak at 731 days, and a minor peak at 975 days. A best-fit analysis in MATHEMATICA, using a form including both a sinusoidal and a linear component, yielded a numerical value for the fading: over the twenty-year span analyzed, the average magnitude of S Per has dropped by about 0.5 magnitude. Possible reasons for this change are discussed in section 5.

Similarly, another analysis was run using only the data before Julian Date 2447000, because earlier papers typically reported only two periods for this time interval. This run very closely reproduced the same peaks in the Fourier spectrum as the entire data set produced, at periods of 748, 790, 948, and 2843 days. These periods are within the expected error, indicating that the most recent data (after JD 2447000) have not contributed any new periods.

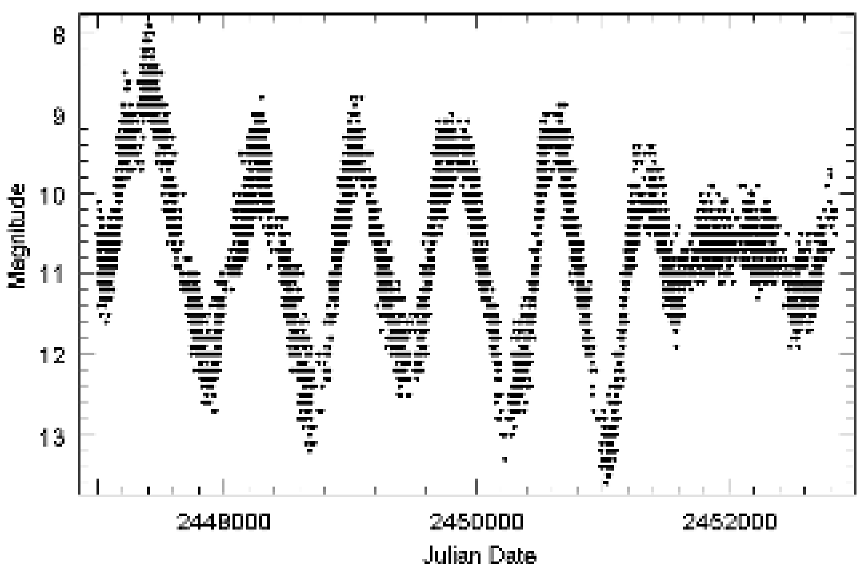

Figure 5. Recent S Persei light curve data.

5. Conclusions

Although the fit is not perfect, it appears as though S Per is a variable that can be qualitatively represented, to this point in time, with three periods and one harmonic. Though this analysis could not take into account the possibility of time-dependent periods or phase shifts, these could very well play a part in the light curve of S Per, and should be studied further. On the other hand, we cannot rule out the possibility that there is only one approximately 800-day period, affected by temporary shifts caused by mechanisms interior to the star, which seems to be the most likely explanation. Considering that 100 years of data are still just the blink of an eye on the time scale of stellar evolution, it is entirely possible that there is only one period, and that during the past 100 years, S Per has seen quite a bit of turbulence.

In this analysis, the 745- and 797-day periods are close to each other, but the depth of the valley between them in the Fourier spectrum indicates that they are distinct, and not the product of aliasing or error. These two periods could be due to one visual period with another period that lags just behind it, perhaps slowed by the stellar envelope. As the energy pulse comes up from the core of the star, a residual pulse is attenuated by the matter it passes through, and shows up as a second, slightly longer, period. This is similar, and may be related, to what is known as the "echo phenomenon" or "overtones" (see, for example, Fadeyev and Muthsam 1992). Alternately, interaction among non-linear modes is possible in S Persei, as discussed by Buchler *et al.* (1996) for the RV Tauri star, R Scuti.

The 952- and 2857-day periods, being multiples, would most likely be due to a physical system inside the star in which harmonics could be expected. Stothers and Leung (1971) postulated that the long periods in variable red supergiants were due to "the convective turnover time of giant convection cells in the stellar envelope." This process could lead to harmonics, if the matter inside the convection cell is not completely cycled in one period, but instead clumps and is moved through in three or four periods. Other possibilities include stellar rotation, or magnetic field effects such as sun spots or pole reversals. These two periods are third multiples; oddly, the second multiple (which would be about 1900 days) did not show a peak in the Fourier spectrum above what statistically was noise. The reason for this is still unknown.

It is interesting to note that after about JD 2447000, S Persei appears to be changing. Two periods seem to have disappeared, and the star's mean magnitude is decreasing. The 731- and 975-day periods found in the recent section of the light curve (see section 4) are most likely related to the original 745- and 952-day periods from section 3 by their proximity, as they are within the calculated error. It does not appear that the star's periods have been changing over time, although an in-depth analysis of that possibility was beyond the scope of this work. Also, the ability of the Fourier transform to find representative periods is based on the number of those periods available in the sample, so the 2857-day period may have disappeared due only to the brevity of the recent data. Most interesting, however, is the gradual fading of S Per's mean magnitude. This may be indicative of long term evolutionary changes, mass loss, or efficiency of dust production in the circumstellar envelope. If the trend continues, it could reflect changes in the star's external opacity which are due to interior variations preceding a possible supernova event, as expected for high mass stars like this one. Knowing this, maser and other observations of S Persei should continue.

6. Acknowledgements

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