# Correlation of Successive Maxima in Mira, $\chi$ Cygni, R Leonis, and R Hydrae

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**Abstract** An analysis was done of the maxima of Mira Ceti, $\chi$ Cygni, R Leonis, and R Hydrae, four Mira-type variable stars chosen from among those studied by Argelander in 1839. A negative correlation between the luminosity of consecutive maxima was found, except for R Hya. $M_{i+1} - M_i$ *vs.* $M_i$ yields a correlation index of $r^2 = 0.6$ for Mira and $\chi$ Cyg, $r^2 = 0.44$ for R Leo, and $r^2 = 0.25$ for R Hya. Data analyzed range from the discovery of Mira (1596) to the present time; they include AAVSO data, and there are from 90 (R Hya) to 179 (Mira Ceti) pairs of consecutive maxima.

## 1. Introduction

In a previous work (Sigismondi, Hoffleit, and Coccioli 2001) we presented a study on all available maxima of Mira Ceti, made in order to ascertain the probability of the occurrence of two successive bright maxima. A negative correlation between $M_{i+1} - M_i$ *vs.* $M_i$ was found for Mira, which suggests that two bright consecutive maxima cannot occur.

I have extended that two-points analysis to three other long-period variable stars, $\chi$ Cyg, R Leo, and R Hya, selected among the first ones studied by Argelander (1839, 1869), and Pickering (1900), for which there is temporal coverage of their light curves greater than a century (see Table 1).

Previous studies on Mira-type variable stars show correlations between the time elapsed between two maxima and their luminosity (Harrington 1965), or an analysis of the fluctuations of the period (Eddington and Plakidis 1929; Percy *et al.* 1999). Monographic studies on those historical variable stars are the book of Guthnik (1901) on Mira Ceti, on its period history Eddington and Plakidis (1929), and Sigismondi, Hoffleit and Coccioli (2002) on the analysis of its maxima and on its discovery; on the period of $\chi$ Cygni Eddington and Plakidis (1929) and Sterken, Broens and Koen (1999); on the period of R Leonis, Hoeppe (1985); and on the period of R Hydrae, Zijlstra, Bedding and Mattei (2002).

## 2. Data set and data analysis

Here is presented the analysis $M_{i+1} - M_i$ *vs.* $M_i$ made on consecutive maxima of the four long period variable stars, Mira Ceti, $\chi$ Cyg, R Leo, and R Hya. The correlation coefficient $r^2$ indicates a strong correlation for Mira Ceti, $\chi$ Cyg, and R Leo, while there is almost no correlation for R Hya (Table 2, Figures 1–4). The four figures show the correlation between the gap in magnitudes between consecutive maxima $M_{i+1}$ and $M_i$ and the magnitude of the maximum $M_i$ for these four stars.

Table 1 gives general information on these stars, with columns 2–3 describing the data studied, columns 4–5 containing mean period and rising time from minimum to maximum from the *General Catalogue of Variable Stars* (Kholopov *et al.* 1998), and columns 6–7 the distance and parallax from the Hipparcos catalogue (Perryman *et al.* 1997).

From the law $y = -ax + b$, where $y = M_{i+1} - M_i$ and $x = M_i$, by definition of absolute magnitude the following equation is valid:

$$y = -a\,M_{abs} + c$$

with $c = b - 5a(\log d - 1)$ and d the distance in parsecs of the star from us.

In Table 3 I report the parameters of the linear fits to the data plotted in Figures 1–4 calculated for absolute magnitudes of the stars.

## 3. Discussion and conclusions

The data are not rigorously homogeneous, since they span over four centuries and were taken by different observers. The first data of Argelander were calibrated by Pickering (1900) using the luminosities of some reference stars indicated by Argelander. I did a similar calibration for the luminosities of Mira Ceti expressed in the Argelander's units as used in Guthnik (1901). Thus Argelander's sample and the data published in Guthnik (1900) for Mira have been made homogeneous with the others. The problem of the scattering of the observations recorded in the early stage of variable star science, and the reliability of a single determination of a maximum's magnitude, remains open.

Müller and Hartwig (1920) gave a list of recorded maxima and minima, with references, ranging from the discovery to 1920. Several astronomers observed those stars, and the observations of a star are not necessarily continued for more than one cycle of variation. Very often they are observed again even after 10 years from the last observation. Since this analysis refers to pairs of consecutive maxima, in Müller and Hartwig (1920) I found data belonging to the same observer and consecutive cycles. When there are many observers, as in the case of Mira, an average was made of the reported magnitudes by all the observers for a given year. All the data have been considered for estimating the error-bar of a single data-point.

The problem of homogeneity is largely reduced after 1905 because Campbell gathered and published all the long period variable maxima/minima data obtained by Harvard and the AAVSO from 1905 to 1949 (1918, 1955). The subsequent AAVSO publication is made with the same criteria of Campbell and covers the years 1949–1975 (Mattei, Mayall, and Waagen 1990). For the last 27 years a similar publication does not exist, but the validated data of AAVSO are available upon request at the AAVSO website (www.aavso.org). I have analyzed those complete light curves as well as the most recent not yet validated ones (2000–2002) (Mattei 2002), with the same program (moving average with a five-day window) in order to have a homogenous sample of consecutive pairs in the last 27 years. Since several observers observed those stars it is fairly easy to cast out some data, which are clearly erroneous due to material errors. The method of evaluation of the maxima for the last 27 years was checked to match the data published for 1949–1975 (Mattei, Mayall, and Waagen 1990) by analyzing with the same method all the data of those stars available online at www.aavso.org upon request.

There is an evident correlation for three of the four stars: Mira Ceti, $\chi$ Cyg, and R Leo, while no correlation has been detected in R Hya. Table 3 suggests also a significant difference in the properties of R Hya from those of Mira, $\chi$ Cyg, and R Leo without having used the correlation index r. In the light curve of R Hya the maxima appear to follow a modulation with a period longer than the cycle period (see Figure 5). In this case I expect a correlation index r ~0 because the differences $M_{i+1} - M_i$, at a given value of $M_i$ are periodically positive and negative, filling the phase-space of $(M_{i+1} - M_i, M_i)$ in a homogeneous pattern.

The comprehension of that phenomenon requires indeed more effort in data analysis either extending to other stars, like R Cyg, as suggested by J. Mattei (2002), or, working with full light curves, performing an autocorrelation analysis, which is impossible with scattered data before 1905 and indeed challenging after 1905. For further information or discussion, the author may be reached at sigismondi@icra.it.

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Table 1. General information on the data of the variable stars studied.

| *Star* | *times of data range* | *pairs of consecutive maxima* | *mean period (days)* | *percent of rising time* | *distance Hipparcos* | *parallax [parsec]* |
|---|---|---|---|---|---|---|
| Mira (o Cet) | 1659–2001 | 179 | 331.96 | 38% | 131 ± 18 | 7.79 [1.07] |
| $\chi$ Cyg | 1686–2002 | 148 | 408.05 | 41% | 108 ± 16 | 9.43 [1.36] |
| R Leo | 1847–2001 | 124 | 309.95 | 43% | 106 ± 22 | 9.87 [2.07] |
| R Hya | 1842–2001 | 90 | 388.87 | 49% | 617 ± 370 | 1.62 [2.43] |

Table 2. Correlation parameters for luminosity of successive maxima.

| *Star* | $r^2$ |
|---|---|
| Mira (o Cet) | 0.60 |
| $\chi$ Cyg | 0.60 |
| R Leo | 0.44 |
| R Hya | 0.25 |

Table 3. Correlation parameters for absolute magnitudes.

| *Star* | *a* | *c [log(absolute luminosity)]* |
|---|---|---|
| Mira (o Cet) | 1.134 ± 0.070 | –2.45 ± 0.34 |
| $\chi$ Cyg | 1.208 ± 0.082 | –0.11 ± 0.48 |
| R Leo | 0.940 ± 0.096 | 0.56 ± 0.48 |
| R Hya | 0.58 ± 0.11 | –2.41 ± 0.34 |

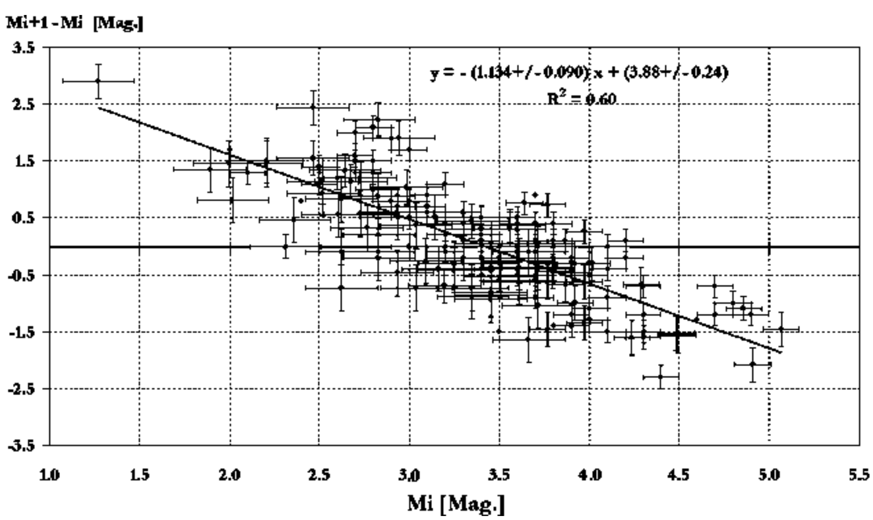


Figure 1. Correlation between the magnitude of a pair of consecutive maxima of Mira Ceti and the visual magnitude of the first maximum of the pair.

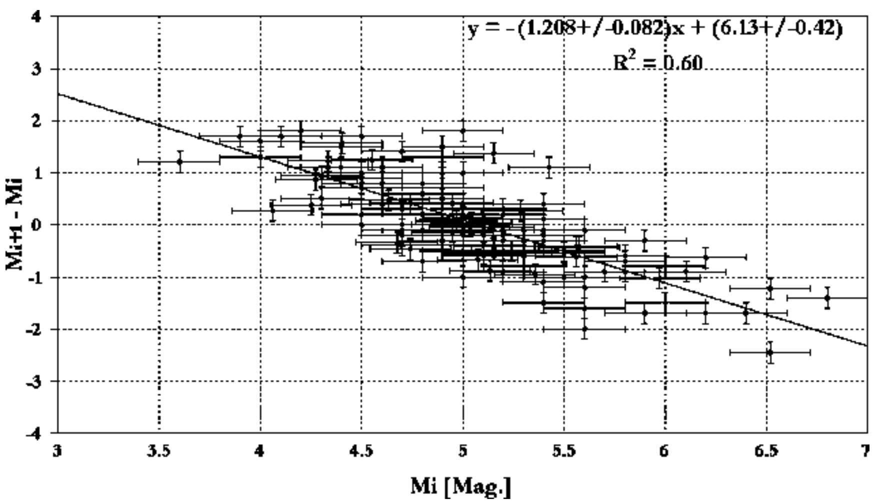


Figure 2. Correlation between the magnitude of a pair of consecutive maxima of Chi Cygni and the visual magnitude of the first maximum of the pair.

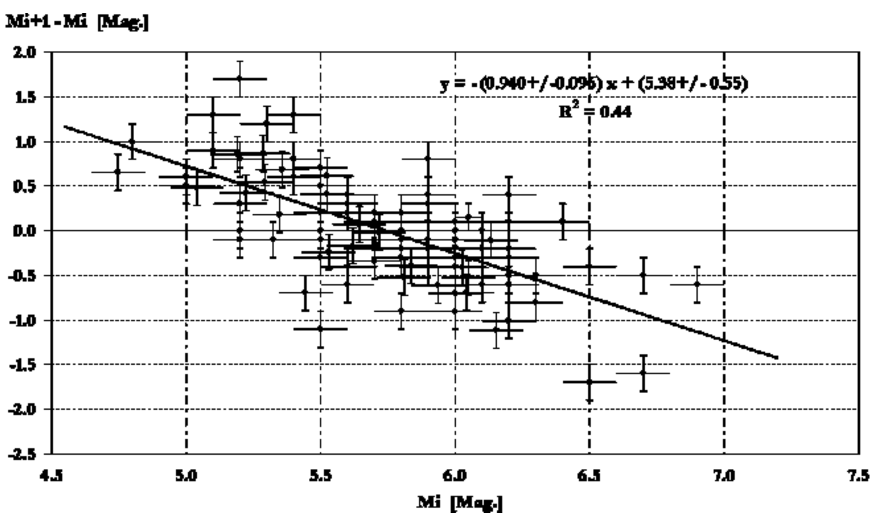


Figure 3. Correlation between the magnitude of a pair of consecutive maxima of R Leonis and the visual magnitude of the first maximum of the pair.

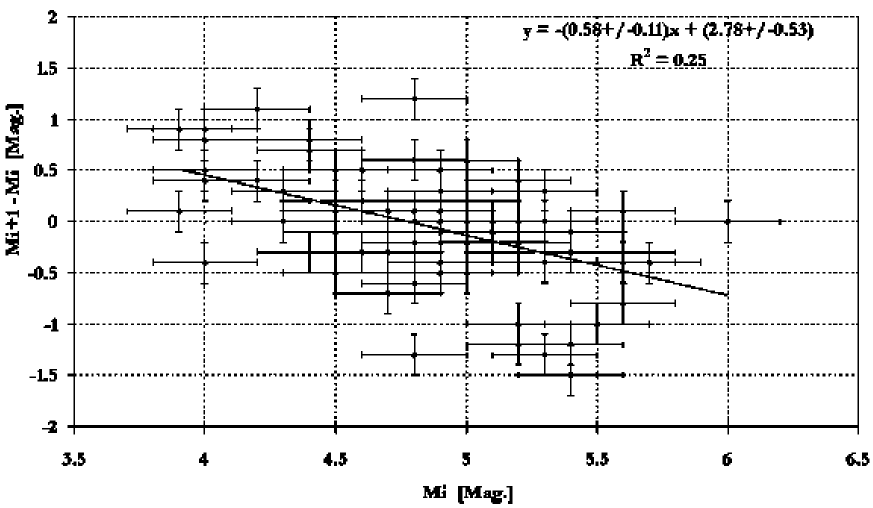


Figure 4. Correlation between the magnitude of a pair of consecutive maxima of R Hydrae and the visual magnitude of the first maximum of the pair.

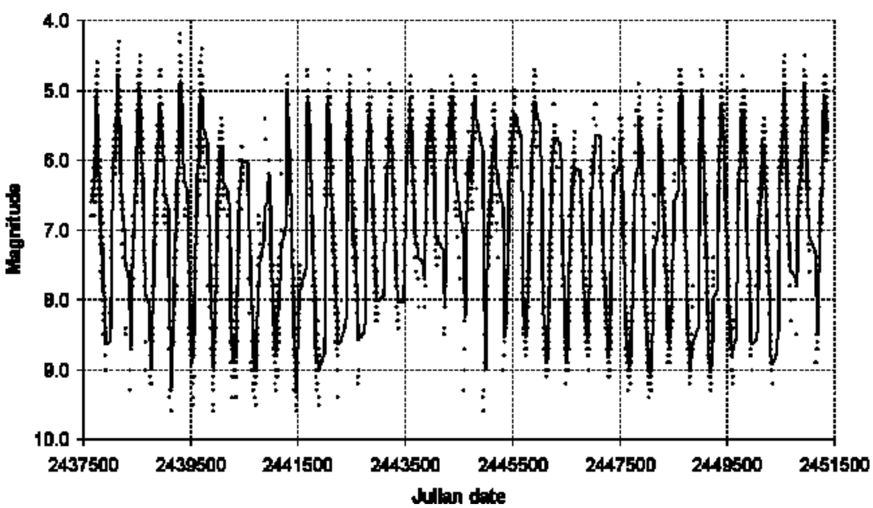


Figure 5. Light curve of R Hydrae 1969–1999 from AAVSO data (dots). The continuous line is a moving average yielding a single point out of 5, for a quick idea of the pattern of the consecutive maxima.