Unsolved Problems in the Evolution of Interacting Binary Stars

Ronald F. Webbink
Department of Astronomy, University of Illinois, 1002 W. Green St., Urbana, IL 61801

Abstract Common envelope evolution is essential to the formation of short-period binaries with compact components. Conditions for its onset, and estimates of its outcome are summarized. However, applied to the well-known binaries V471 Tauri and T Coronae Borealis, these conditions lead to serious inconsistencies with their observed properties.

1. Introduction

One of the most important processes in the evolution of interacting binary stars is common envelope evolution (for a review, see Iben and Livio 1993). By this term, we identify a process by which one component of a binary (in the following discussion the primary component) completely engulfs its companion star (the secondary). The secondary thus becomes a second core within the common envelope, along with the core of the primary. These two cores spiral toward each other, a process which dissipates the orbital energy and angular momentum of the embedded binary in its surrounding envelope. The orbital energy available may suffice to unbind the surrounding envelope from the embedded cores; if not, they will continue spiraling inward until the less dense core is tidally disrupted. Short-period binaries in which one or both components are compact (white dwarfs, neutron stars, or black holes) are now believed to be the products of common envelope evolution. Cataclysmic variables and low-mass X-ray binaries are apparently products of this process.

The concept of common envelope evolution was introduced by Paczyński and Ostriker in 1975 at a symposium on the evolution of close binary stars (Paczyn´ski 1976). They were motivated in part by evolutionary questions elicited by the well-known binary V471 Tauri. V471 Tau is a white dwarf-red dwarf eclipsing binary member of the Hyades, with so small an orbital separation that it would fit easily within one of the red giants in that cluster. Stellar evolution provides that the white dwarfs as massive as that in V471 Tau are incubated in the cores of even more extended giants. The conclusion that a binary can survive engulfment follows inevitably from this circumstance.

In the following discussion, I will identify circumstances in which common envelope evolution may occur, describe how one may deduce the properties of a binary which emerges from common envelope evolution, and then examine two well-known close binaries, the self-same V471 Tauri mentioned above, and the recurrent nova T Coronae Borealis, which would both seem likely products of this evolutionary channel. As we shall see, certain peculiarities in their properties seem to defy basic physical principles (conservation of mass and energy), suggesting that important processes at play in common envelope evolution remain to be identified.

2. Onset of common envelope evolution

2.1 Dynamical instability

Three basic time scales characterize the structure and evolution of stars. In an interacting binary, these same time scales are manifested in the response of the donor star when it loses mass to its companion. The removal of mass produces an immediate drop in pressure throughout the interior of the star, as the weight of that mass is lifted from the stellar surface. This pressure drop is transmitted through the star at the speed of sound, i.e., on a dynamical time scale, resulting in an adiabatic expansion throughout the interior as the star attempts to restore hydrostatic equilibrium. This expansion may or may not produce a wholesale expansion of the stellar radius, depending on the degree to which mass lost from the stellar surface is replaced at the stellar surface by material of lesser or greater density. Adiabatic expansion reduces both temperature and density locally; in the deep interior of the star, it therefore tends to quench nuclear burning, which is extremely temperature-sensitive. In these regions, energy losses due to diffusion (or convection) are no longer resupplied by burning, and they are compressed under the weight of overlying layers until that compression restores nuclear burning into balance with energy losses, the star reaching a new state of thermal equilibrium. The time scale characterizing this process of internal energy redistribution is the amount of time it would take a star to radiate away the thermal energy of its interior, i.e., a thermal time scale. In reaching this new state of thermal equilibrium, stars tend to expand overall unless they are nearly homogeneous in composition, that is, unless they lie on the main sequence. On longer time scales, of course, the chemical profile inside a star evolves through a succession of nuclear burning cycles, i.e., on a nuclear time scale. Phases of stable nuclear burning are characterized typically by concomitant expansion of the star, punctuated by short periods of contraction when new burning cycles are ignited at the center of the star.

The time scale on which mass transfer occurs in an interacting binary depends on orbital dynamics as well as on the hierarchy of time scales characterizing the stellar response to mass loss. The more massive a donor star is in relation to its companion, the more dramatic will be the contraction of its tidal (or Roche) lobe as mass transfer proceeds. Because more massive stars evolve more rapidly, the first instance of mass transfer in an interacting binary is invariably triggered by overflow of the more massive component. In general, one finds that such stars with moderately deep surface convection zones cannot satisfy hydrostatic equilibrium while remaining within their Roche lobes. These stars are unstable to mass transfer on a dynamical time scale, which is comparable with the orbital period at the onset of mass transfer. The consequence is rapid engulfment of the companion in common envelope evolution. In contrast, stars with deep radiative envelopes tend to be unstable to thermal time-scale mass transfer, unless they are much more massive than their companions (in which case they may evolve into dynamical instability) or they are scarcely more massive (in which case they may transfer mass on a nuclear time scale).

The pattern which emerges is that stars on the giant branch and asymptotic giant branch form the principal source of common envelope systems. Their cores, stripped of their envelopes, become the white dwarf components of cataclysmic variables, or, if sufficiently massive, may evolve to core collapse, and become the neutron star components of low-mass X-ray binaries. The very large factors by which stars grow in radius on giant and asymptotic giant branches creates a correspondingly large window for this evolutionary channel.

2.2 Secular instability

Darwin (1879) showed that under extreme circumstances, a binary system in synchronous rotation is secularly unstable in the presence of tidal dissipation. Tidal exchange of angular momentum into rotation of a star from a companion orbiting about it will of increase the rotation rate of the star. The angular momentum drawn from the orbit leads to a contraction of the orbit and, through Kepler's third law, to an increase in the orbital angular frequency as well. The converse holds for angular momentum exchange in the opposite direction (as in the Earth-Moon system). Tides will extract angular momentum from the higher frequency element (orbit or rotation) and transfer it to the less rapidly rotating element. Ordinarily, rotational angular momenta of the stellar components of a binary are very small compared with the orbital momentum, and stellar rotation is brought into co-rotation with the orbit with only a minor change to the orbit. However, if one component is much more massive than its companion, and nearly fills its Roche lobe, it may take more angular momentum to spin it up to synchronism than is available from the companion orbit. In this case, the companion spirals in toward the massive star, spinning it up, but orbiting even more rapidly, until tides drain so much angular momentum from its orbit that it enters the envelope of the more massive star.

3. Outcomes of common envelope evolution

The physics of common envelope evolution are extremely complex and interrelated in detail; a complete model is not yet within the grasp of current computational techniques. However, it is possible to constrain possible outcomes significantly from simple energy considerations. Common envelope evolution extracts energy (E_{orb}) from the binary orbit,

$$E_{orb} = -\frac{GM_1 M_2}{2A}, \tag{1}$$

and deposits it into the common envelope. At the onset of common envelope evolution, that envelope is bound to the primary component which just fills its Roche lobe; all of the energy needed to expel it is presumed to come from gravitational sources, that is, from the gravitational binding energy of the orbit. The initial binding energy of the envelope can be approximated as

$$E_{env} \approx -\frac{GM_1(M_1-M_c)}{R_1}. \tag{2}$$

The radius of the primary, R_1, is at this instant equal to the radius of its Roche lobe, Ar_L, where r_L is a function only of the binary mass ratio, M_1/M_2, A is the binary separation, as above, and G is Newton's gravitational constant. Equating E_{env} to a fraction, α_{CE}, of the orbital energy difference between initial and final state, and recalling that the lobe-filling component is stripped in the process from its initial mass, M_1, to its core mass, M_c, we obtain the relation (Webbink 1984)

$$-\frac{GM_1(M_1-M_c)}{A_i r_L} = \alpha_{CE}\left(-\frac{GM_cM_2}{2A_f}+\frac{GM_1M_2}{2A_i}\right). \tag{3}$$

Figure 1 summarizes the conditions under which giant and asymptotic giant branch stars are expected to be unstable due to dynamical time-scale mass transfer or to secular tidal instability, and the degree of orbital contraction upon emergence from common envelope evolution, on the assumption that energy conversion is perfectly efficient ($\alpha_{CE}=1$).

The threshold shown here for dynamical instability is based on simple stellar models consisting of a point mass core with an isentropic ideal gas envelope (Hjellming and Webbink 1987), and that for secular instability on the approximation that such stars have moments of inertia $I_1 = 0.205\,(M_1 - M_c)\,R_1^2$. Lobe-filling stars with relatively massive cores may be stable against dynamical time-scale mass transfer, but, for reasons outlined below, stars with such tenuous envelopes are unlikely to reach a lobe-filling state. Under any other conditions, stars which first fill their Roche lobes as giants or asymptotic giant branch stars enter common envelope evolution, emerging only after very substantial reductions in binary separation and orbital period.

4. The curious case of V471 Tauri

The membership of V471 Tau in the Hyades provides important constraints on its prior evolution, constraints which are not available in other post-common-envelope systems. Because it is a well-detached binary—the K dwarf component is only ~70% the radius of its Roche lobe (e.g., İbanoğlu 1978)—the cooling age of its white dwarf component should measure the difference between the current cluster age and the age of the white dwarf progenitor at emergence from common envelope evolution. Because common envelope evolution itself must be very brief (to suppress radiative loss of the dissipated orbital energy), this is negligibly different from the age of the primary star when it first filled its Roche lobe, and we should then be able to extract a rather precise estimate of its mass at that time.

In reality, the properties of the white dwarf in V471 Tau defy so straightforward an analysis. The Hyades contain a number of other single white dwarfs, and at least one other white dwarf-red dwarf binary, HZ 9. Table 1 summarizes the effective temperatures, angular diameters, and masses (deduced from fitting these quantities to cooling curves) of the single white dwarf members, drawn from Weidemann *et al.* (1992), and of the white dwarf components of the binaries V471 Tau and HZ 9.

For these binary components, effective temperatures have been adopted from Bois *et al.* (1988) and from Guinan and Sion (1984), respectively, and the same distance modulus adopted to the Hyades as used by Weidemann *et al.* White dwarfs decrease in radius with increasing mass (neglecting finite temperature effects), and decrease in temperature with increasing age. It is therefore a surprise to find that V471 Tau appears to contain both the most massive white dwarf in the Hyades and the hottest (and therefore youngest), this despite the fact that common envelope evolution should have cut off growth of the degenerate core that became this white dwarf before its single contemporaries had shed their own envelopes. Put another way, it appears that single stars in the Hyades manage to shed their envelopes down to a mean core mass of $\sim 0.63\ M_\odot$ without intervention of a companion, while V471 Tau has subsequently contrived to grow a more massive core from a less massive progenitor in spite of having core growth arrested prematurely by mass transfer. Evidently the history of V471 Tau is more complicated, and it may well be a much older post-common-envelope system than it first appears.

The single white dwarfs in the Hyades provide another important insight into the circumstances under which stars may enter common envelope evolution. Their mean mass is scarcely greater than the core masses of their progenitor stars, which Weidemann *et al.* estimate to have been $2.5\ M_\odot$ at birth, when those stars reached a thin double-shell-burning configuration on the asymptotic giant branch. The helium-burning shells in such stars undergo thermal pulses, and the rapid attenuation of further core growth implied by the Hyades white dwarfs provides strong support for identification of such a stage with the onset of a "superwind" from such giants (see Willson 2006). In a binary system, mass loss in a stellar wind leads to expansion of the binary orbit in inverse proportion to the remaining mass of the binary; when the rate of erosion of the more massive component much exceeds its rate of expansion by nuclear evolution, as is the case during the superwind phase, that star's Roche lobe expands more rapidly than the star itself does, and it can no longer initiate common envelope evolution (Iben and Tutukov 1985).

5. The stranger case of T Coronae Borealis

T CrB is a well-known recurrent nova which presents another interesting dilemma for common envelope evolution. Unlike the great majority of cataclysmic variables, it has a long orbital period, 227.5 days. Nevertheless, this orbital period is far too small for this binary to have eluded past mass transfer, if the hot component is a white dwarf. The spectroscopic orbit of the M3 III donor star is now very well-determined (Kenyon and Garcia 1986). An orbit was also determined for the hot component by Kraft (1958), but in combination with the orbit of the M3 III star, and the apparent absence of eclipses, it yields a lower mass limit of 2.3 $M_\odot$ for the hot component, impossibly large for a white dwarf. This large mass was a motivating factor in this author's proposal (Webbink 1976) that outbursts of this object are accretion events onto main sequence stars, rather than more conventional thermonuclear runaways on a white dwarf. Because of the difficulty in measuring the hot component orbit, and the unreasonable mass obtained from it, that orbit is widely regarded with great skepticism (e.g., Selvelli *et al.* 1992). For purposes of this discussion, then, let us assume that the hot component is a white dwarf of 1.4 $M_\odot$, and that the system only barely misses eclipsing ($i = 68°$); the mass deduced for the donor is then 1.31 $M_\odot$. This combination of parameters presents the most favorable case possible for surviving common envelope evolution at a long orbital period with a white dwarf component.

The stability constraints on common envelope evolution outlined above and illustrated in Figure 1 suggest that the progenitors of 1.4 $M_\odot$ white dwarfs must exceed 1.97 $M_\odot$ at the onset of mass transfer, in order to be unstable to common envelope formation. Stellar evolution puts an upper mass limit of ~12 $M_\odot$ on the white dwarf progenitor at this stage. For this range of donor masses, the simple mapping of initial to final orbital separations illustrated in Figure 1 then yields final orbital periods ranging from 700 days (for a 1.97 $M_\odot$ donor) down to 0.45 day (for a 12 $M_\odot$ donor), for an assumed ejection efficiency $\alpha_{CE} = 1$. (I adopt the $R_1(M_1, M_c)$ relations given by Politano 1996.) The long orbital period of T CrB, however, can be accommodated only if the donor was no more massive than 2.5 $M_\odot$. Stars in the progenitor mass range, from 1.97 $M_\odot$ to 2.5 $M_\odot$, for T CrB then lie well beyond the onset of the superwind phase, and are retreating from their Roche lobes, rather than approaching them, as inferred above from the masses of white dwarfs in the Hyades. In fact, only the most massive progenitors allowed can reach their Roche lobes before the onset of a superwind, and so only the short-period extreme, up to $P \approx 1$ day, of the range of post-common-envelope orbits delimited above can be populated. Such short periods are consistent with the orbits of other recurrent novae such as U Sco and V394 CrA, but it would appear that common envelope evolution as now understood is incapable of accounting for the properties of T CrB.

6. Conclusion

Common envelope evolution is a phenomenon now firmly entrenched in the repertoire of close binary evolution. It provides a sound, if not yet calculable, physical explanation for the origin of short-period binaries with compact components. Ironically, however, attempts to understand the origin of two well-known examples of such systems, V471 Tau and T CrB, present major inconsistencies, suggesting that important physical processes in common envelope evolution have yet to be identified.

7. Acknowledgements

I am grateful to the AAVSO for the invitation to participate in this meeting, the Astrophysical Theory Centre at the Australian National University for its hospitality during the writing of this paper, and Walid El-Khoury and Michael Burgess for help in its preparation. This work was supported in part by National Science Foundation grants AST92-18074 and AST96-18462.

References

Bois, B., Lanning, H. H., and Mochnacki, S. W. 1988, *Astron. J.*, **96**, 157.

Darwin, G. H. 1879, *Proc. Roy. Soc. London*, **29**, 168.

Guinan, E., and Sion, E. 1984, *Astron. J.*, **89**, 1252.

Hjellming, M. S., and Webbink, R. F. 1987, *Astrophys. J.*, **318**, 794.

İbanoğlu, C. 1978, *Astrophys. Space Sci.*, **56**, 219.

Iben, I., Jr., and Livio, M. 1993, *Publ. Astron. Soc. Pacific*, **105**, 1373.

Iben, I., Jr., and Tutukov, A. V. 1985, *Astrophys. J., Suppl. Ser.*, **58**, 661.

Kenyon, S. J., and Garcia, M. R. 1986, *Astron. J.*, **91**, 125.

Kraft, R. P. 1958, *Astrophys. J.*, **127**, 620.

Paczyński, B. 1976, in *Structure and Evolution of Close Binary Systems*, IAU Symp. 73, eds. P. Eggleton, S. Mitton, and J. Whelan, Reidel, Dordrecht, 75.

Politano, M. 1996, *Astrophys. J.*, **465**, 338.

Selvelli, P. L., Cassatella, A., and Gilmozzi, R. 1992, *Astrophys. J.*, **393**, 289.

Webbink, R. F. 1976, *Nature*, **262**, 271.

Webbink, R. F. 1984, *Astrophys. J.*, **277**, 355.

Weidemann, V., Jordan, S., Iben, I., Jr., and Casertano, S. 1992, *Astron. J.*, **104**, 1876.

Willson, L. A. 2006, *J. Amer. Assoc. Var. Star Obs.*, ***35**, 62.*

Table 1. Hyades white dwarfs.

*Name**	T_{eff}(*K*)	θ_d (μ*as*)	M ($M_\odot$)
EG 26	14500	1.48	0.66
EG 36	19700	1.19	0.62
HZ 9	20000:	1.01:	0.69:
EG 39	21200	1.24	0.63
EG 37	26000	1.05	0.65
EG 42	27700	1.10	0.61
V471 Tau	35000	0.98	0.78

**Boldface entries identify close binary systems.*

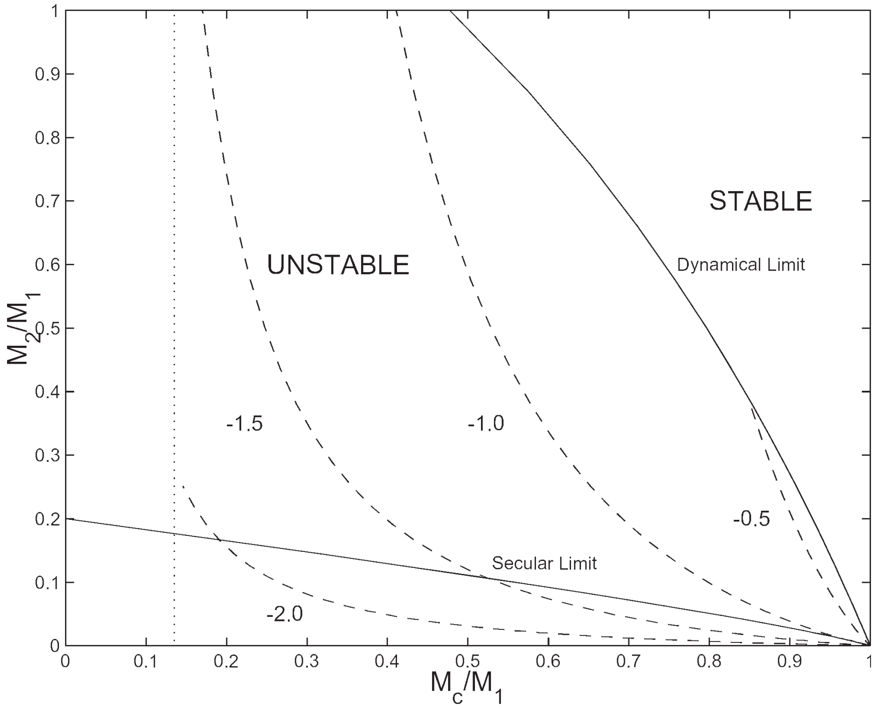

Figure 1. The primary core mass-secondary star mass plane. Masses are normalized to the mass of the lobe-filling primary star at the onset of mass transfer. Solid lines mark the thresholds below which stars with degenerate cores (red giants and asymptotic giant branch stars) are unstable due to dynamical time-scale mass transfer (upper curve) and to a secular tidal instability (lower curve). The vertical dotted line marks the Schönberg-Chandrasekhar limit, the core mass which stars exhaust on the main sequence; this is the smallest degenerate core mass physically allowed. Dashed lines mark the ratio of final to initial orbital separations for binaries undergoing common envelope evolution, and are labeled by the common logarithm of that ratio, $\log(A_f / A_i)$.