What Makes a Star Act Like a Mira?—and Other Conclusions From Dynamical Atmosphere Models

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Abstract This review stresses four ways that pulsation affects the atmosphere of a star with consequences for the appearance of the star. These are:

- Motion in the form of standing waves, traveling waves, and shocks.
- Extension or levitation of the atmosphere.
- Heating and cooling of the gas.
- Mass loss.

The motion is directly observable (at least in some parts of the atmosphere), and the extension of the atmosphere also produces observable consequences. The heating and cooling result in part from compression and expansion of the gas, which keep the low-density gases of the atmosphere out of radiative equilibrium. In Miras, at least, mass loss is driven by a combination of high pressure produced by shock-heated gas, momentum imparted by shocks, and radiative forces on dust grains. Cooling by expansion between shocks provides an opportunity for the formation of dust to occur close to the star.

All numerical model results presented in this paper are by G. H. Bowen.

1. Motion and the importance of boundary conditions

Most of the discussion that follows is based on detailed study of pulsation in Mira variables. My goal in this review is to stress insights that are likely to be relevant in all cases of stellar pulsation. However, it is well to remember that the discussion is based on detailed modeling for one particular case, and we are likely to find additional phenomena to be of interest in other classes of stars. There are several special properties of Miras that are relevant to this discussion. (I) Miras are low-surface-gravity giants with high luminosities. This gives them large atmospheric scale heights (even in the static approximation), low escape velocity, and the ability to drive a massive wind with radiation pressure. (II) Miras are (predominantly if not entirely) radial pulsators, and most (if not all) are pulsating in the fundamental radial mode. (See Maeder's review in this volume for a description of these modes.) (III) Miras have low effective temperatures (around 3000K), giving them an emergent energy spectrum peaking around 1 micron. Striking features of Mira spectra are (i) deep absorption bands of molecules and (ii) strong emission lines (hydrogen in the visible, MgII in the ultraviolet) that are present during just a portion of the pulsation cycle. (IV) Mass loss from Miras plays a key role in stellar evolution, terminating the "asymptotic giant branch" phase.

Traditionally, pulsation has been studied with stellar interior models that have

relatively simple boundary conditions applied at the "surface," the photosphere. (The photosphere is defined as the location from which photons have a 50% chance of escaping. This defines the radius of the star, technically. For most classes of stars, the radius is well-defined this way, whether one chooses a limited wavelength region around the peak, or chooses a suitable average of opacity. For Miras, this is more difficult, as I will discuss further in section 2.) The periods deduced from these models are then adopted in the construction of models for the effects of pulsation on the atmosphere of the star. The standard assumption of interior modeling is that there is complete reflection of all waves at the surface, since there is assumed to be no medium to carry the wave energy out from there. In reality, an extended atmosphere can absorb and dissipate considerable pulsation energy, at least for relatively short pulsation periods. The atmospheric models usually assume that the periods are not greatly altered by what happens in the atmosphere, and most atmospheric models make use of the periods from analyses that are strictly valid only for relatively small internal pulsation amplitudes. An important future step will be the construction of models that include the relevant non-linear physics of both the driving zone and the atmosphere. However, very useful insights into the interplay between the interior and the atmosphere have already been derived from the latest generation of dynamical model atmospheres (Bowen 1990):

> For short pulsation periods (P), near the apparent surface of the star (the photosphere), traveling waves form, steepen, and grow into shocks (Figure 1). There is no obvious region of standing waves, and no evidence for reflection at the photosphere.
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> For long pulsation periods, standing waves form below the photosphere (Figure 2). These typically grow to large amplitude, because there is strong reflection in the region a little outside the photosphere. It is not total reflection, however; there is sufficient energy transmission to permit the formation of fairly strong traveling shocks.

The above result can be understood in terms of the importance of the "acoustic cutoff period," defined at each point by:

$$P_{acoustic} = 4 \pi H / c_{sound} \quad (1)$$

where H is the pressure scale height (defined by $(1/P)\,(dP/dr) = (-1/H)$) and c_{sound} is the speed of sound. Waves with periods less than $P_{acoustic}$ travel freely through the atmosphere, while waves with longer periods than $P_{acoustic}$ die out quickly. With standard expressions for isothermal H and c_{sound}, Equation 1 gives $P_{acoustic} \propto T^{1/2}\, r^2$. Well inside the photosphere, T is very high; it decreases rapidly with r near the photosphere. Thus $P_{acoustic}$ also decreases with increasing r near the photosphere. Farther out, the temperature levels out; then the r^2 and term dominates, and $P_{acoustic}$ increases with increasing distance from the star. The result of all this is that there is a minimum value of $P_{acoustic}$ somewhere near the photosphere, and this minimum

value determines which waves can escape easily into the atmosphere. For Miras, the fundamental mode period is longer than the minimum in $P_{acoustic}$ in the fully dynamic models, while the first overtone period much closer to the minimum in $P_{acoustic}$. As a result, reflection of waves for fundamental mode models is strong, while for overtone modes it is quite weak.

Observable phenomena may result from the physics of $P_{acoustic}$. When $P > 2P_{acoustic(min)}$, the dynamical models show multiple shocks per period, with the number of shocks per period increasing with $P / P_{acoustic(min)}$. These can give rise to bumps on the light curve or even the appearance of pulsation with two periods. For examples and a more complete discussion of the role of $P_{acoustic}$ in Miras, see Bowen (1990).

2. Extension of the atmosphere ("levitation")

Waves traveling down a density gradient grow and steepen into shocks. The steeper the gradient, the larger the amplitude these shocks reach at a given position in the atmosphere. The density gradient for a static atmosphere is very steep. Waves traveling down this gradient rapidly reach very large amplitudes, amplitudes that are inconsistent with periodic motion. Considering an extreme case illustrates why this occurs: If each passing shock imparts an outward velocity υ_o to the material, and between shocks the only force acting is gravity, then the material returns to the same position with inward velocity $-\upsilon_o$ and after a time given by (Hill and Willson 1979; Willson and Hill 1979):

$$\upsilon_{esc}\, P / 2r = 38.3\, Q^d = (\beta\ / (1 - \beta^2)) + (1 - \beta^2)^{-3/2} \arcsin(\beta) \qquad (2)$$

where $\beta = \upsilon_o^{periodic} / \upsilon_{escape}$, $\upsilon_{escape} = \sqrt{2GM} / r$, and Q^d is the "pulsation constant in days = $P^d\sqrt{\rho/\rho_\odot}$. (Note that the "pulsation constant" isn't really a constant. For Miras, Q varies from about 0.07 to 0.11 day in models pulsating in the fundamental mode, and for other classes of stars for low-order radial modes it typically falls between 0.01 and 0.1 days. It allows a very approximate guess about the likely pulsation period of a star when nothing else is known, which is very convenient, but it is also sometimes overemphasized in the discussions of stellar pulsation. An alternative expression for Q is $P\ (M / M_\odot)^{1/2}\ (R / R_\odot)^{-3/2}$.) In this, ρ is the mean density of the star (its mass divided by its volume) and $\rho_\odot$ is the mean density of the Sun. The shock velocity amplitude, $\Delta\upsilon$, is $2\upsilon_o$ in this ballistic limit. It is important to recognize that even in the ballistic approximation given by Equation 2 the maximum $\Delta\upsilon$ is substantially less than $g\,P$ for fundamental mode pulsation in Miras. There are quite a few examples in the literature of attempts to measure the photospheric value for g using some variation on $\Delta\upsilon = g\,P$; these underestimate g, and have led to erroneous mode identifications for Miras.

If the density decline is too steep, the shocks will become too large, and will push material out with too high speed ($\upsilon_o > \upsilon_o^{periodic}$). In this situation, the material does not have time to return to its initial position before the next shock hits it. This puts

more material farther out, and makes the density gradient less steep. If the density gradient is not steep enough, the shocks will stay small, and v_o will ultimately be too small. Material will end up closer to the star and so the density gradient will steepen. Thus, the density adjusts until the material is just able (under the influence of gravity and any other forces acting) to return to its initial position after one period *P*. The result is an extension of the atmosphere, with higher density than in the static case throughout the region where the shocks are reaching maximum amplitude.

Note: What dominates the appearance of $\rho(r)$ in Figure 3 is actually a second effect that takes over when the material is no longer moving in a strictly periodic manner, but is migrating outward as part of the general mass loss process. The velocity profile begins to approach a constant outflow with moderate-amplitude waves superimposed, and the density in response becomes approximately $1/r^2$— see section 4. The density adjustment described above actually applies over a narrow range of *r*, but affects the mass loss rate by its effect on the value of ρ where the outflow begins to dominate.

The ballistic analysis above makes two simplifying assumptions that are usually not correct for the Miras. One assumption is that the pressure forces are negligible except very close to the shock front; the other is that the motion consists of a single ballistic trajectory per cycle. From Figure 2 and the discussion in section 1 is it clear that this second assumption is almost never valid when the star shows large amplitude pulsation, since in that case $P > 2\ P_{acoustic}$ and multiple shocks are formed per cycle. In the case where multiple shocks are present one can still derive an *upper limit* on the shock amplitude from Equation 4, but it is quite a bit more generous (thus less useful) in such cases.

Observable consequences of the atmospheric extension are two: First, it is very difficult to measure the size of a Mira. The apparent surface at a given wavelength is the point where photons have a roughly 50:50 chance of escaping from the atmosphere. For a given wavelength, λ, the 50% escape locus is given roughly by

$$\tau\lambda = \int_R^\infty X^2\, \kappa\lambda \rho dr = 1 \qquad (3)$$

where $X = R/r < 1$, $\tau\lambda$ is the optical depth, $\kappa\lambda$ is the absorption coefficient (units cm^2/gm), and ρ is the density. For wavelengths that have a relatively high probability of absorption (large $\kappa\lambda$), this "surface" may appear at several times the radius derived from standard interior modeling (determined using a wavelength-averaged absorption coefficient and, generally, calculated without taking into consideration the extension of the atmosphere seen in Figure 3). Second, this extension may be part of the reason for the large visual amplitudes of Miras. As (very opaque) molecules like TiO form in the extended atmosphere, and are later broken up by passing shocks, their now-you-see-it-now-you-don't opacity in the visual can make big changes in the visual magnitude from one phase to another.

3. Heating and cooling

How a refrigerator works (simplified slightly): Gas is compressed and therefore it heats up. The compressed gas loses heat to the cooler room air. The gas expands and therefore it cools. The cooled gas absorbs heat from the interior of the refrigerator.

Shocks in a stellar atmosphere produce entirely the same effect: Compression heats the gas. Energy is radiated away; the gas approaches the local radiative equilibrium temperature. Gas expands between shocks and thus cools, in some cases below the radiative equilibrium temperature. Such over-cooled gas absorbs energy from the radiation field and thus approaches the radiative equilibrium temperature. If the density is low, then the exchange of energy with the radiation field is inefficient, and the gas makes big excursions in temperature away from the radiative equilibrium values.

Two extreme cases are seen in the models (Figure 4 a, b). The "calorisphere" case (Figure 4a) shows up in models with low mass, relatively low luminosity, higher effective temperature, and/or low abundances of the dust-forming elements. Models with high mass and luminosity and with metal abundances like that of the Sun or higher tend to look more like Figure 4b. For details, see papers by Bowen (1988) and Willson (2000).

4. Mass loss

The mass loss rate, $\dot{M}$, for a steady wind is

$$\dot{M} = (4\pi r^2)\, \rho \upsilon \qquad (4)$$

where ρ is the density and υ is the outward speed at position r in the atmosphere. Conservation of mass, sometimes called the "continuity equation," requires this to be the same everywhere for a steady mass loss. By the arguments in section 2, high in the outer atmosphere of a pulsating star the density is *much* larger than it would be in a static atmosphere. That is one of the reasons why pulsating stars easily lose mass. The outward speed at large distances from the star, υ_∞, is determined mainly by the process that provides the final push. In Miras and other pulsating giants this may be:

A. The formation of a high-temperature region, a calorisphere. This allows a steep pressure decline, together with the action of many small shocks, to push the wind out.

or

B. The formation of dust grains and the force of the starlight pushing these away from the star. The dust grains collide with the gas and so drag it out, too.

The formation of dust grains is made possible by (a) the extension of the atmosphere

and (b) the refrigeration of key portions of it (section 3). Once formed, the grains can survive the passage of shocks in the very low-density region where they spend the rest of their time as they leave the star. In case b the dust-driven expansion leads to further cooling of the atmosphere. (See Bowen and Willson 1991; Willson, Struck, and Bowen 1997).

5. Conclusions and implications

All the above suggest the identification of the "Mira phenomenon" as occurring for very luminous red giant stars as they reach a stage of devastating mass loss. This realization allows some further observational comparisons: Models predict mass loss rates that are very sensitive to the stellar luminosity (*L*), radius (*R*), and mass (*M*). This means that selecting for the Mira phenomenon, or for the highest mass loss rates commonly observed, will select for stars with a small range of L and R at a given M. (See Willson 1997, 2000 or Willson, Bowen, and Struck 1996 for further discussion of this point and figures illustrating the fit to the observational data.) The result produces an excellent fit to both the observed Mira period-luminosity relation and the empirical mass loss relation discovered by Reimers (1975). Both of these fits force a major change in the interpretation of the observed relations (Willson, Bowen, and Struck 1996).

In summary: The effects of pulsation on the stellar atmosphere include (often complex) motions; extension of the atmosphere, so that the star appears larger in some colors; heating and/or cooling, giving a temperature pattern that is quite different from that in a static atmosphere; and mass loss resulting from the extension and the heating/cooling as they contribute to some final "driving" mechanism. Detailed models show how these effects are linked to observable—or defining!—properties of the Mira variables: large visual amplitudes, complex light-curves, emission lines (from post-shock cooling), diameters that depend on how you look at them, and heavy mass loss.

6. Addendum, 2006

Two more recent papers covering material relevant to this topic are: Willson (2000) and Willson and Kim (2004).

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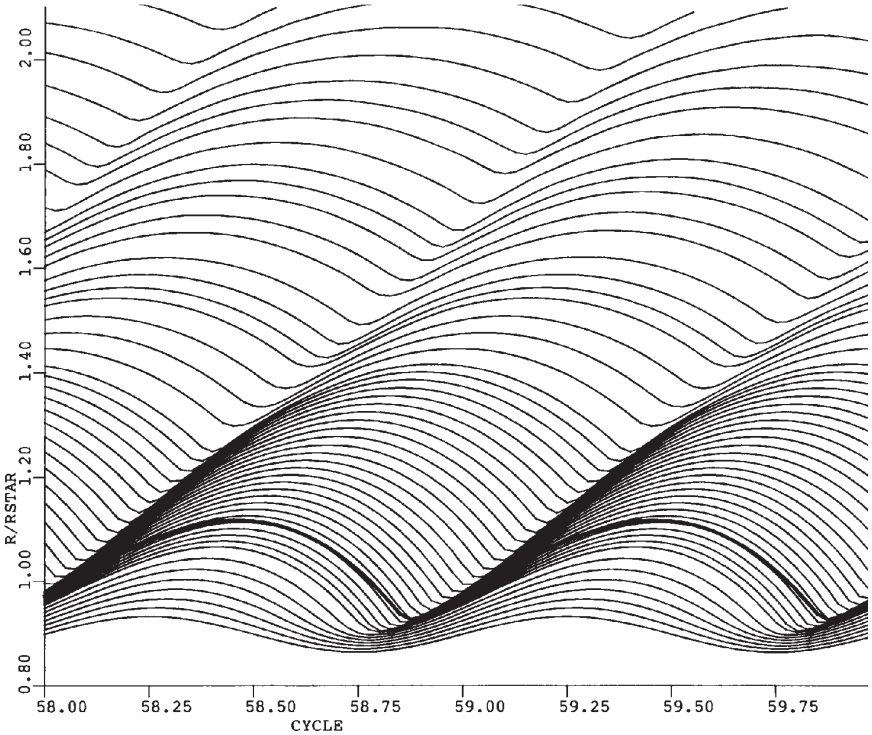

Figure 1. Radius vs. time for a model driven with a short period, corresponding to pulsation in the first radial overtone mode. Where the motion shifts abruptly from infall to outward motion a shock is formed. The photosphere is indicated by the heavy line; the wave motion does not change character at the photosphere, indicating weak (if any) reflection of waves at the "surface" of the star. From Bowen 1990.

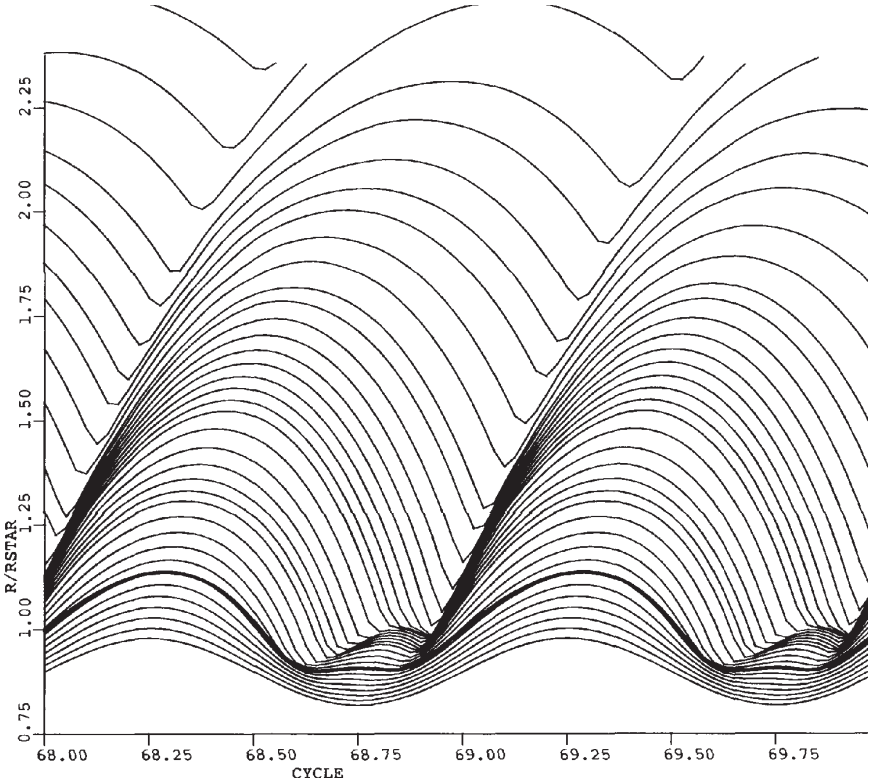

Figure 2. Radius vs. time for a model driven at a longer period than in Figure 1 (corresponding to fundamental radial mode of pulsation). Note the change from a standing wave to traveling wave at about the level of the photosphere. For this case, with $P/P_{acoustic} > 2$, two shocks are formed each cycle. In a range of similar models it is found that either shock may dominate the observed light curve (by having larger amplitude and producing more radiation). From Bowen 1990.

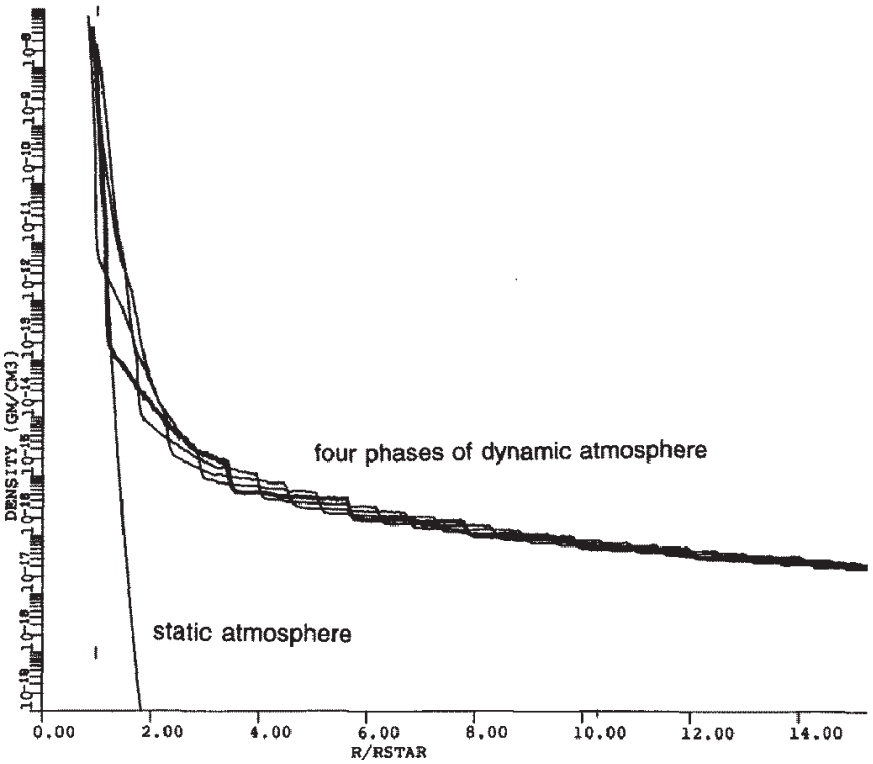

Figure 3. Density as a function of height in the atmosphere from a fundamental mode Mira model by Bowen. In the inner atmosphere there is still the steep decline found for static models, but the outer atmosphere and outflowing wind regions have much higher densities than a static model for the same star.

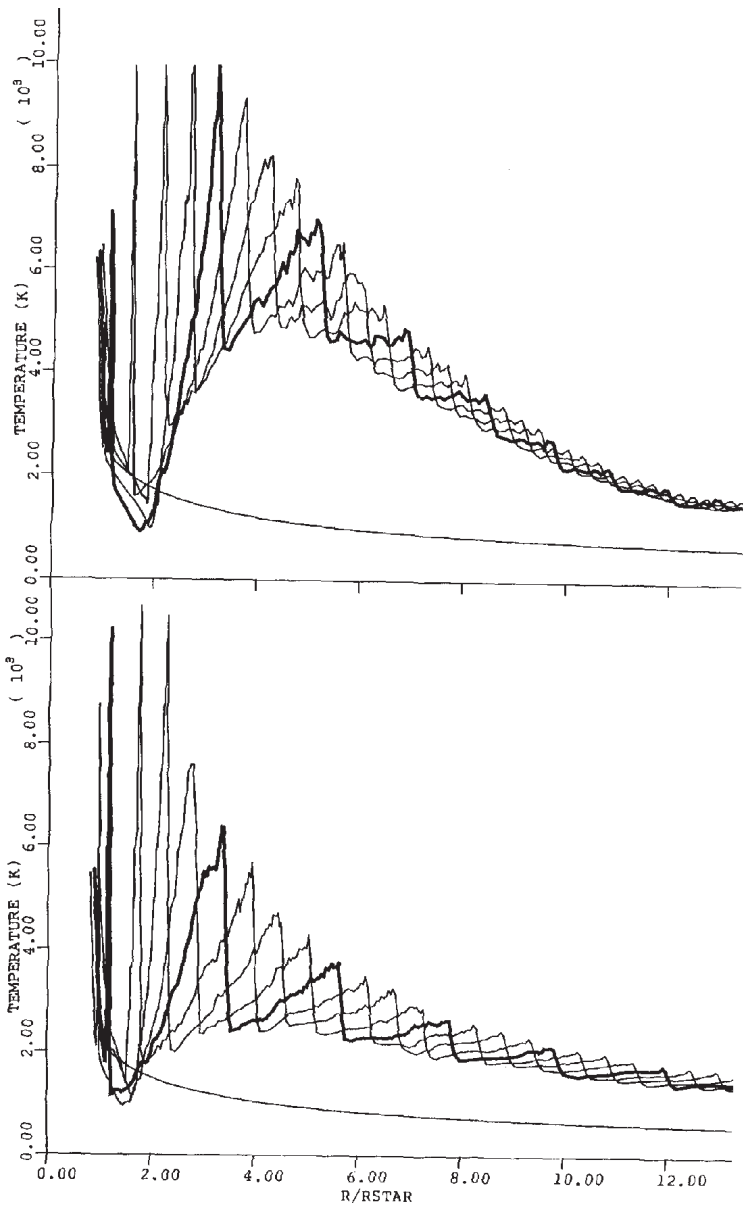

Figure 4. Temperature vs. height (bottom) for two fundamental mode Mira models by Bowen. 4a (top): Model with a "calorisphere." 4b (bottom): Model in which dust is forming. The pressure of the starlight on the dust pushes it outward, and it drags the gas along, creating a rapid outflow. The gas expands as it flows away from the star, and this expansion cools it, reducing or eliminating the "calorisphere."