A Search for Random Cycle-to-Cycle Period Fluctuations in Five δ Scuti, SX Phoenicis, and RR Lyrae Variables

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Abstract Random cycle-to-cycle period fluctuations in pulsating stars were first studied by Eddington and Plakidis in 1929, and have subsequently been found in almost all Mira stars, many RV Tauri stars, and a few Cepheids. They complicate the use of (O–C) diagrams for studying evolutionary period changes in these stars. We set out to study such fluctuations in four δ Scuti stars and SX Phoenicis stars; we added one RR Lyrae star for testing purposes. We discuss complications that arise in applying this method to short-period stars. Only one of the five stars—the SX Phoenicis star XX Cyg—showed statistically-significant fluctuations.

1. Introduction

δ Scuti stars are short-period pulsating variable stars, of spectral type A5 to F2, lying on or near the main sequence in the Hertzsprung-Russell diagram. They are among the most numerous variables among the naked-eye stars. Many have complex light curves which are a result of the simultaneous presence of two or more radial or non-radial pulsation modes. SX Phoenicis variables are closely-related stars; they are old disc or Population II stars, whereas δ Scuti stars are Population I. See Breger (1979) for an excellent introduction to the topic or, at a higher level, Breger (2000).

Period changes in pulsating variable stars can, in some cases, reflect evolutionary changes in the star, and can therefore be used to test models of stellar evolution. This represents the only direct observation of the normal, slow evolution of stars. The period changes are studied by the (O–C) method (Sterken 2005). If the period of the star changes at a constant rate, as it would (to first order) due to evolution, then the (O–C) diagram would be a parabola. If the period of the star changes abruptly, then the (O–C) diagram would be a broken straight line. In actual fact, the (O–C) diagrams of δ Scuti and SX Phoenicis stars are complex and, even though many of them have been studied for decades, the interpretation of the diagrams is ambiguous. Parabolas can be force-fit to the diagrams, but the resulting magnitudes and signs of the period changes do not always agree with evolutionary predictions. See Templeton (2005) for an excellent recent review of period changes in these

stars. Among other things, this review describes the long history of attempts to explain the period changes in δ Scuti stars in terms of evolutionary changes.

2. Random cycle-to-cycle period fluctuations

One process that can lead to non-parabolic (O–C) diagrams is random cycle-to-cycle period fluctuations. Eddington and Plakidis (1929) developed a formalism for studying these, and showed that they occurred in a few Mira stars. Percy and Colivas (1999), using an AAVSO database of times of maximum and minimum brightness of almost 400 Mira stars, over seventy-five years, showed that almost all of their (O–C) diagrams were dominated by this effect. Furthermore, the (O–C) diagrams of many RV Tauri and SRd variables are strongly affected by this effect (Percy *et al.* 1997), and it even occurs in at least one Population II Cepheid (RU Cam: Percy and Hale 1998) and at least one Population I Cepheid (SV Vul: Turner *et al.* 2001), but not in most other Cepheids (Turner and Berdnikov 2001). Nor was it present in BW Vul, a β Cephei star (Percy *et al.* 2003). The ratio ε/P is 0.01 to 0.05 in Mira stars, and 0.005 to 0.02 in RV Tauri and SRd stars, where ε is the average fluctuation per cycle, measured in days.

In this paper, we test the hypothesis that some of the complexity in the (O–C) diagrams of δ Scuti and SX Phoenicis stars is due to this effect. Furthermore, if δ Scuti and SX Phoenicis stars are physically different, it is possible that the effect occurs in one (most likely the lower-gravity SX Phoenicis stars) but not the other.

3. Data

Several δ Scuti or SX Phoenicis stars have been studied for decades, and dozens (or more) of times of maximum recorded. Unfortunately the data are usually very heterogeneous, often including photographic or visual data, as well as photoelectric data for the larger-amplitude stars. The photoelectric or CCD observations may be made using different instruments, through different filters; the time of maximum varies slightly, depending on the wavelength of observation. The Eddington-Plakidis algorithm requires that there be measured times of maximum that are only a few (preferably 1–10) cycles apart, and this is not often the case for short-period variables. In fact, since the periods of these stars are a few hours, it may not be possible to measure times that are a few cycles apart from one location, because daylight interferes.

We searched the literature for collections of times of maximum which were as numerous, well-distributed, and accurate as possible. By the process of elimination, we arrived at the following four stars.

3.1. CY Aqr

CY Aqr is an SX Phoenicis star, *V* range 10.42–11.16 (Rodríguez *et al.* 2000; *General Catalogue of Variable Stars* (GCVS) Kholopov *et al.* 1985). Many

interpretations of its (O–C) diagram have been published, most of them postulating a series of abrupt period changes. Powell *et al.* (1995) provide the most complete account, and we have used 472 times of maximum, expressed to ± 0.0001 day, provided by their paper.

3.2. VZ Cnc

VZ Cnc is a δ Scuti star, *V* range 7.18–7.91 (Rodríguez *et al.* 2000; GCVS), which was discovered in 1950. Fu and Jiang (1999) have published an (O–C) diagram which suggests that there is no long-term period change. We have used the 138 times of maximum, expressed to ± 0.0001 day, provided by their paper.

3.3. XX Cyg

XX Cyg is an SX Phoenicis star, *V* range 11.28–12.13 (Rodríguez *et al.* 2000; GCVS). Blake *et al.* (2002) have published a comprehensive study which, like earlier studies, clearly shows an increasing period; they derive a rate of period change of $+5.64 \times 10^{-13}$ day/cycle. We have used 138 times of maximum, expressed to ± 0.001 day, provided by their paper.

3.4. DY Peg

DY Peg is an SX Phoenicis star, *V* range 11.28–12.13 (Rodríguez *et al.* 2000; GCVS). The most recent and comprehensive study is by Hintz *et al.* (2004). This and previous studies strongly and consistently show a decreasing period, the rate of period change being about -6×10^{-12} day/day. We have used 277 times of maximum, expressed to ± 0.0001 day, provided by their paper.

After the analysis of these four stars had been completed (by PC), KB did further work in developing a new computer program, and studying the effect of assumptions about the weighting of data points (see below). We therefore added one more test star to the sample, XZ Dra.

3.5. XZ Dra

XZ Dra is an RRab star, *V* range 9.59–10.65 (GCVS). We used 504 times of maximum, expressed to ± 0.001 to 0.0001 day, from the GEOS database at: http://dbrr.ast.obs-mip.fr/maxRR.html.

The periods (in days) and the epochs (HJD) that were used to determine the (O–C) values for CY Aqr, VZ Cnc, XX Cyg, DY Peg, and XZ Dra were: 0.061038328 and 2426159.485, 0.178363665 and 2431550.7187, 0.13486513 and 2430671.1015, 0.072926308 and 2429193.4506, 0.476497 and 2441928.374, respectively.

4. Analysis

To test for random cycle-to-cycle period fluctuations, the hypothesis is made (Eddington and Plakidis 1929) that some or all of the (O–C) variations are due to random fluctuations (ε) in period from one cycle to the next. There are also random

errors (α) in the measured times of maximum light. These errors are each assumed to be accidental and uncorrelated. We define $z(r)$ as the (O–C) of the rth maximum, compared with the ephemeris, and $u(x,r) = z(r + x) – z(r)$ is the accumulated delay in x periods which, according to the hypothesis, is the sum of x uncorrelated random fluctuations. Allowing also for the random errors in the measured times of maximum light, we find that the average value $< u(x) >$ over all values of r is given by

$$< u(x) >^2 = 2\alpha^2 + x\varepsilon^2 \quad (1)$$

so a plot of $< u(x) >^2$ against x should be a straight line with slope ε^2 and intercept $2\alpha^2$. Note that x is measured in cycles.

We therefore first determined the $z(r)$ from the best available ephemeris of each star, and then used these to determine $< u(x) >^2$ for all possible maxima, x cycles apart. The values of $< u(x) >^2$ were plotted against x. Normally, the region between $x = 0$ and $x = 20$ can be used to determine the slope and intercept.

In practice, it is often better to determine α from the value of $u(1)$, which is usually better defined than the intercept.

There are several ways to implement this algorithm. We have initially done it within EXCEL. One of us (KB) has also written a suite of programs in C++ and a manual, which we would be pleased to share; we plan to eventually make it available on the website:

http://www.astro.utoronto.ca/~percy/index.html

There is a complication, in this work, which was not present in the analysis of Mira stars. In the case of the Mira stars, every time of maximum brightness has been recorded for many decades, covering dozens of cycles, thanks to the work of AAVSO observers. The value of ε is determined from the values of $< u(x) >^2$ for x between 1 and 20. Each of those values has approximately the same weight, because each of them is determined from the differences between several dozen values of $z(r)$—especially for small x. In the case of the δ Scuti stars, however, there are many values of x for which $< u(x) >^2$ is not determined, or is determined from only one or two $z(r)$ differences, and therefore has little or no weight. This can be seen from the error bars in the points in the figures. One alternative is to weight the points according to the number of $\Delta z(r)$ values contributing to it. A second is to weight the points inversely proportional to the standard error of each point. Ideally, the results should not depend crucially on which weight is used. If it does, the result should be regarded with suspicion.

Figure 1 shows the $< u(x) >$ diagram for about thirty cycles in Mira, showing how the error bars are related to the number of pairs of (O–C)s that contribute to each point. Figures 2 and 3 show the $< u(x) >$ diagrams for CY Aqr and XX Cyg, as examples.

5. Results

5.1. CY Aqr

The value of ε is 0.00096 with a 3-σ upper limit of 0.0014 and a 3-σ lower limit of zero; the 3-σ upper limit of ε/P is 0.0225 (Figure 2). Here and throughout, σ represents the standard deviation. The value of α as determined from the intercept is 0.0028 day but the value as determined from u_1 is 0.007 day. The value of ε is *almost* significantly non-zero at the 3-σ level—but not quite.

5.2. VZ Cnc

The value of ε is 0.00026 with a 3-σ upper limit of 0.0064 and a 3-σ lower limit of zero; the 3-σ upper limit of ε/P is 0.0064. The value of α as determined from the intercept is 0.0042 day; the value as determined from u_1 is consistent with this. The value of ε is thus not significantly non-zero at the 3-σ level.

5.3. XX Cyg

The value of ε is 0.000279 with a 3-σ upper limit of 0.00033 and a 3-σ lower limit of 0.00021 (Figure 3); the 3-σ upper limit of ε / P is 0.0020. The value of α as determined from the intercept is 0.00041 day but the value as determined from u_1 is 0.0007 day. This is the only star in our sample which shows the Eddington-Plakidis effect at the 3-σ level.

5.4. DY Peg

The value of ε^2 is negative, so the nominal value of ε is zero, with a 3-σ upper limit of 0.000063; the 3-σ upper limit of ε/P is 0.00086. There are no significant random cycle-to-cycle period fluctuations. The value of α as determined from the intercept is 0.0007 day, which is consistent with the value as determined from u_1.

5.5. XY Dra

The value of ε is 0.0021 ± 0.041 which is not significantly different from zero. The value of α as determined from the intercept is 0.023 ± 0.029 day.

6. Discussion

The application of the Eddington-Plakidis algorithm is much more complicated in these short-period stars in which only a small fraction of times of maximum have been measured, and in which consecutive times of maximum are rarely measured. We have chosen variables with several hundred measured times of maximum; it will not be practical to analyze most other short-period variables.

One additional possible source of scatter in the (O–C) diagram in these stars is multiperiodicity: many δ Scuti stars have two or more periods. Of the four that we have studied, only VZ Cnc is considered to be multiperiodic (Fu and Jiang 1999), and there is nothing unusual in the $< u(x) >$ diagram for this star.

Nevertheless, we have shown that, in the five stars analyzed, random cycle-to-cycle period fluctuations are not large, and are therefore not a dominant contributor to the (O–C) diagrams of these stars. The values of ε/P are: for CY Aqr, 0.0157 ± 0.015; for VZ Cnc, 0.0015 ± 0.002; for XX Cyg, 0.0021 ± 0.0001; for DY Peg, 0.0013 ± 0.0007. Only the value for XX Cyg is significant to the 3-σ level.

The cause of the complexities in the (O–C) diagrams of these stars, i.e., the non-parabolic shape, remains unclear.

7. Conclusions

Random cycle-to-cycle period fluctuations, if they are present in δ Scuti and SX Phoenicis stars, are considerably smaller than they are in Mira stars and RV Tauri stars (ε / P ≤ 0.002 as opposed to 0.005 to 0.05) and are not a significant contributor to the complex (O–C) diagrams of these stars.

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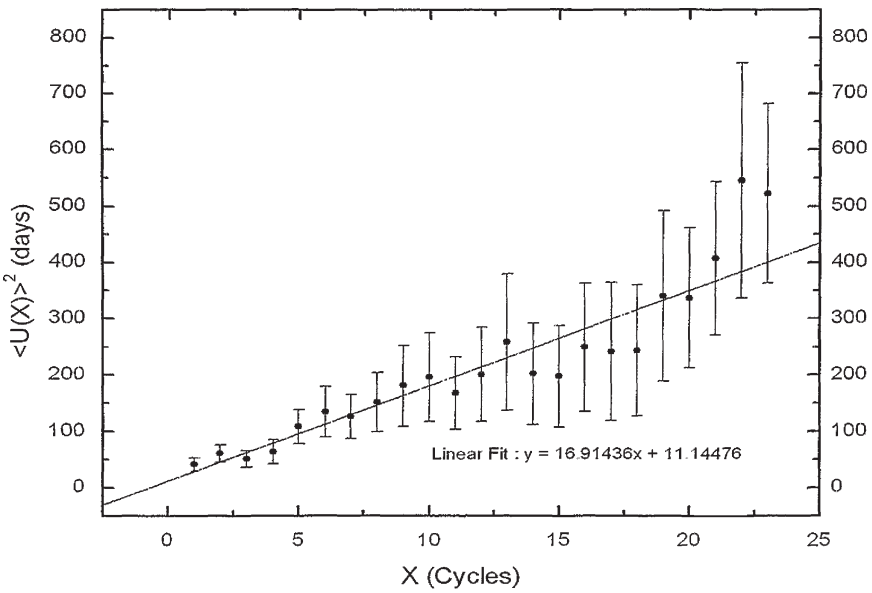

Figure 1. Graph of $< u(x) >^2$ vs. x of o Ceti (weight = sample size). The $< u(x) >$ diagram for about thirty cycles of Mira. Note that the error bars become larger as x becomes larger, because there are fewer and fewer pairs of (O–C)s that are x cycles apart.

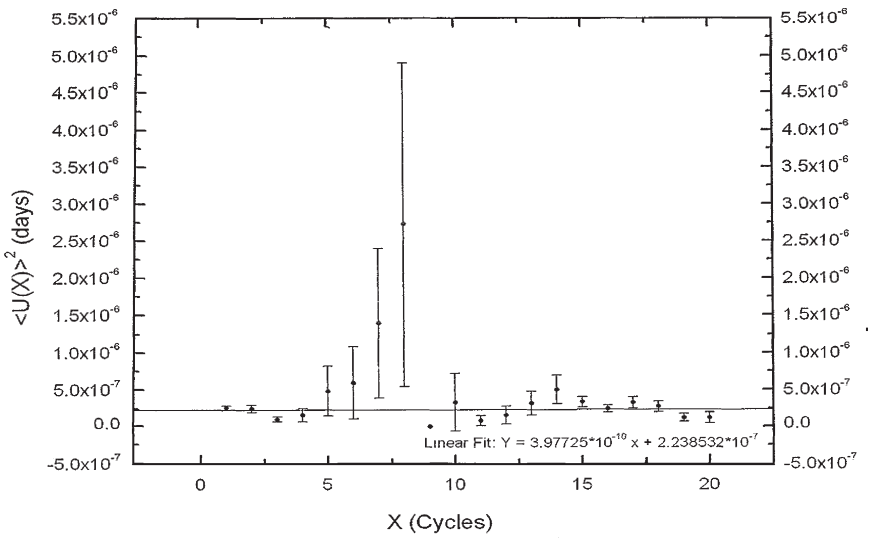

Figure 2. The < $u(x)$ > diagram for CY Aqr. The best-fit line is almost horizontal, indicating that the cycle-to-cycle period fluctuations (represented by ε) are negligible.

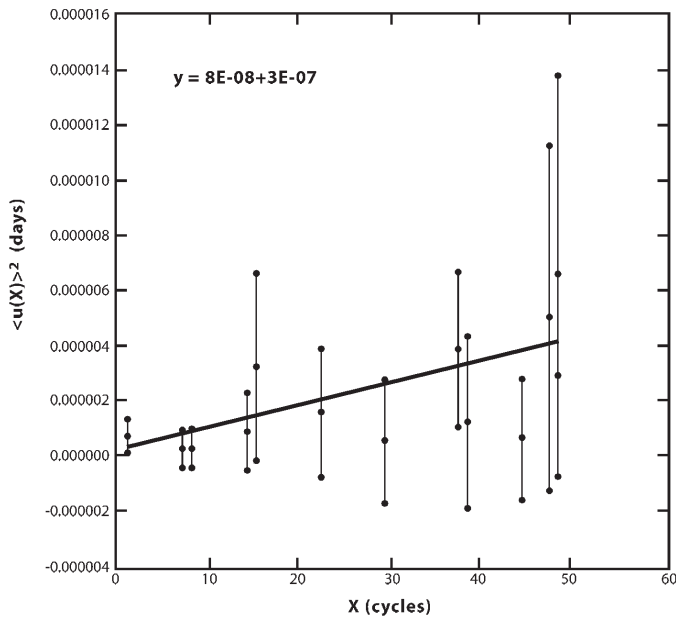

Figure 3. The < $u(x)$ > diagram for XX Cyg. The best-fit line has a positive slope that is significant at the 3-σ (standard deviation) level, indicating that the cycle-to-cycle period fluctuations (represented by ε) are present and significant.